Chapter 4:
Development and Application of a Numerical Model of the Formation and Evolution of the Perseid Meteoroid stream[2]

4.1 Introduction

The recovery of comet 109P/Swift-Tuttle in 1992 marked the beginning of an intensive effort to characterize one of the largest known Earth-crossing bodies. Much has been learned of Swift-Tuttle in the intervening years (cf. Yau *et al.*, 1994), but the comet's equally famous trail of meteoroidal debris remains mysterious. The return of the comet was presaged by a strong increase in activity from the Perseids beginning most notably in 1991 (Brown and Rendtel, 1996). This marked the first occasion when a large change in the flux of the shower was unambiguously recorded. Indeed, Olivier (1925) comments that "...the Perseids appear with no remarkable variations in numbers practically every August".

The Perseid shower has been recognized in the sky almost as long as records of such phenomena have been kept. Hasegawa (1993) has traced ancient records of the

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stream back 2000 years and it seems probable that the stream is older still. Detailed observational histories of the stream have been given by Kronk (1988) and Rendtel *et al.* (1995). The shower is also notable as the first instance in which a comet was definitively linked to a meteor shower, this connection having been made by Schiaparelli (1867).

The first attempts to understand the stream in an analytical form were those of Twining (1862), who investigated the perturbing effects of the Earth on Perseid meteoroids and found no sensible perturbations from this mechanism. Further research through the late 19th and early 20th century concentrated on interpreting visual observations of the shower. Throughout this period, there was general understanding that comets and meteoroid streams were linked, the weight of opinion being that the latter originated from the former, but contrary views were not uncommon. Whether meteoroids were continually discharged or periodically released from comets remained unclear.

That progress in understanding the stream relied heavily on the untangling of the cometary - meteoroid decay process is highlighted by Guigay (1947) who postulated that the stream was formed entirely by a collision between a proto-Swift-Tuttle and another body. The resulting spall accounted for the Perseids and at least five other comets noted by Guigay to have relatively close orbital intersections. Kresak (1957) pointed out the numerous difficulties in this interpretation and its contradiction to the mounting photographic meteor data then available for the stream.

Hamid (1951) was the first to model the ejection of the meteoroids from Swift-Tuttle using Whipple's (1951) “icy-snowball” cometary model and to analytically follow the resulting orbits under the effects of secular planetary perturbations. He noted that the formation and subsequent evolution of the stream is intimately linked with the past history of the comet, which he determined through secular perturbations of the then best available orbit for Swift-Tuttle. The variation in orbital elements for Perseid meteoroids was found to be in general agreement with photographic data, assuming ejection velocities of order 10 m/s, and the age of the stream was determined to be 40 000 years.

Southworth (1963) performed a more detailed analysis of the evolution of the stream by computing numerically the gravitational perturbations on individual stream meteoroids instead of mean perturbations from secular theory. He found that the variation observed in the radiant position and velocities of meteoroids in the stream implied scattering much stronger than planetary perturbations alone could explain. Using similar ejection velocities as Hamid, he concluded that either strong non-gravitational effects out of the orbital plane were at work or the stream was formed not through gradual disintegration of the parent comet but rather by way of a single, large cometary explosion (citing Guigay's (1947) hypothesis) approximately 1000 years ago. His work implied an upper limit of 6000 years for the age of the stream.

Sekanina (1974) investigated the dynamics of the Perseid stream based on a detailed consideration of the likely ejection conditions from the parent comet and the effects that variations in these conditions, such as location and direction of ejection, might make on the final meteoroid distributions. By examining ancient records of recorded appearances of the Perseids, he concluded that a systematic variation in the time of recorded Perseid returns relative to the perihelion passage of the comet suggested that the meteoroid emission lasted for several months, probably beginning shortly before perihelion process and implicitly assumed to be nearly continuous during this time. In particular, he suggested that the comet may vary its dust output dramatically from apparition to apparition, resulting in preferential locations for strong Perseid returns relative to the comet's perihelion passage and the initial emission epoch.

The concept that the Perseid stream was formed by emission of meteoroids at a single location along the orbit of Swift-Tuttle was further developed by Katasev and Kulikova (1975). Using a variety of ejection locations and velocities, they determined that the best agreement between computed orbits from an isotropically emitting Swift-Tuttle and the observed stream was found using velocities of 100 m/s and an ejection centered at 30° true anomaly. No account of subsequent planetary perturbations or the past history of the comet was employed and the fit relied entirely on the veracity of the orbital elements for the stream presented by Southworth (1963).

The failure of Swift-Tuttle to return in 1981, as predicted based on the 1862 orbital solution alone (cf. Marsden, 1973), was the most significant development in the understanding of the stream to that time. It became clear that our ideas about Swift-Tuttle based on these observations of the comet alone were in error and along with them previous attempts to understand the stream. The recovery of 109P/Swift-Tuttle early in 1992, and its subsequent perihelion in December of that year, provided hope that serious attempts to understand the stream might be successful as the complete history of Swift-Tuttle's orbital evolution over the last two thousand years was then possible.

Wu and Williams (1993) have used Whipple's ejection model in conjunction with a Monte Carlo approach to model the behavior of 500 test meteoroids of the same mass ejected during the 1862 passage of Swift-Tuttle. They conclude that gravitational perturbations from the planets move the original non-Earth intersecting orbits into Earth crossing paths and suggest that much of the recent intense activity from the Perseids is from 1862 ejecta, with 1994 being the culmination of this activity. The use of small numbers of test particles of only one mass and an older orbit for Swift-Tuttle limit the generality of their results. To improve on this early model Williams and Wu (1994) used a better orbit for the comet and a distribution of masses to make quantitative predictions concerning activity for the Perseids in the early 1990's as well as locations for the maximum of the shower in each year from 1988-1995. The results still suggested that peak activity would occur in 1994, but the predicted times for maximum were consistently two hours earlier than observed.

Harris and Hughes (1995) have investigated the distribution of semi-major axes of photographic Perseid meteoroids. They find no variation as a function of mass and conclude that the final ejection velocities for Perseid meteoroids are independent of mass and all of relatively high velocity. This result will be discussed in detail in Sect. 4.2. Harris *et al.* (1995) expanded upon this result by modelling the ejection of Perseids using a Maxwellian velocity distribution centered about 0.6 km/s. Through integration of 109P/Swift-Tuttle backward for 0.16 Ma, they also simulated formation of the stream as a whole, taking ejections from the comet every 5000 years without accounting for planetary perturbations or radiation forces. They conclude that the stream is roughly 160 000 years old.

Here we develop a detailed numerical model for the formation and subsequent evolution of the Perseid stream. From our analysis we will attempt to gain some understanding of several key questions, such as:

1. How do the initial ejection conditions assumed affect the final observed distributions and over what time scales are the initial ejection conditions “erased” due to radiation forces and planetary perturbations? In particular, are the final distributions sensitive to the assumed cone angle over which ejections take place, the largest distance from the sun the meteoroids are ejected and the assumed density of the meteoroids? What changes in the final distributions is a function of mass? What is the best model representation of the ejection process? What is the range of initial ejection velocities?

2. Why has the position of the outburst peak of the Perseids observed over the last decade changed position in the stream? Why has the outburst portion of the stream also varied in intensity so much in this time interval? Why did this recent outburst activity “turn-on” so quickly in 1991? What are the underlying causes of the outbursts - intrinsic changes in the dust output of the comet in the past, the recent passage of Swift-Tuttle or some other effect?

3. What ejection(s) contribute most to the outburst activity we have seen in the stream over the last decade? Are most of these meteoroids from the 1862 passage of the comet as has been widely assumed?

4. What is the age of the main core of the Perseid stream? What is the ultimate age of the stream?

5. What is the current progression rate of the node of the stream?

6. What effect does the Earth have on the longer-term development of the stream?

7. What are the mechanisms, which remove meteoroids from the stream and over what time-scales do they act?

8. What controls the delivery of Perseid meteoroids to Earth?

4.2 Initial Conditions: The Cometary Decay Process

4.2.1 Physical models

Stream meteoroids are ejected from comets. As comets approach the sun, the number of meteoroids ejected from a comet tends to increase, as does the magnitude of the ejection velocity. The ejection velocity is a small fraction of the orbital velocity of the comet and hence the daughter meteoroids move along similar orbits to the parent comet. Sublimating volatiles (primarily water-ice) are responsible for release of the particles through momentum exchange with the meteoroid grains.

The preceding paragraph summarizes those general aspects of the meteoroid ejection process for which there is near unanimous agreement by workers in the field. Adding additional details to the preceding picture, particularly quantitative ones, requires interpretation of often contradictory observational and theoretical aspects of the cometary ejection process. Remarkable as it seems, this picture is almost identical to the one first presented by Whipple (1951). The only major change from that early model which might be widely accepted today is the observational fact that the active regions of comets (and hence the areas where meteoroids might be ejected) are small fractions of the total surface area of the comet and thus dust is initially confined to collimated jets immediately after leaving the nucleus surface (cf. McDonnell *et al.* 1987). At great distances from the nucleus, however, the meteoroids in such jets tend to spread out into larger cones and the final physical picture may not be very different from Whipple's (cf. Jones, 1995).

To try to model the evolution of a meteoroid stream, the process by which the stream initially formed is of considerable interest. Whether the formation process is the dominant evolutionary process (in comparison to planetary perturbations or radiation forces) is not clear and may vary from stream to stream. Since uncertainty exists about the formation process, we choose to use several different models of formation along with wide variations for those parameters, which we feel, are particularly poorly known in order to determine just how strongly the initial conditions affect the final results. In the end, each model and set of parameter choices lead to a range of possible values for one crucial number; namely the final ejection velocity of the meteoroid relative to the comet. Knowing this value along with the location of ejection, comet orbit and meteoroid shape permits forward integration of the equations of motion for the stream meteoroid and some approximate estimate of its future location.

As it is impossible to make a rigorous determination of the precise location of ejection for a meteoroid, a Monte Carlo approach must be employed. Here we assume that meteoroids are ejected at random values of true anomaly over the arc of the 109P/Swift-Tuttle's orbit inside 4 A.U. in numbers proportional to the amount of solar energy received by the nucleus. That meteoroids would be ejected with equal probability for all values of true anomaly (for 109P, between $233° < \nu < 127°$) under these assumptions was first noted by Kresak (1976). This result is due to the r^{-2} variation of solar flux and the $r^2\, d\nu/dt$ constant of motion from Kepler's second law removing the effects of changes in ν on the meteoroid production function. That ejection occurs inside 4 A.U. for 109P/Swift-Tuttle has been constrained partially by the observations of Boehnhardt *et al.* (1996) and O' Ceallaigh (1995) who observed little or no coma in Swift-Tuttle at 5 A.U. during its 1992 apparition. While water production is usually taken to cease near 3 A.U. (cf Festou *et al.* (1993)), some more distant production is commonly observed in many comets and we choose 4 A.U. as a compromise, acknowledging that much of this distant production is due to compounds more volatile than water, with the dust-gas interaction dynamics likely to be quite different. We will investigate the effects on the observed stream of choosing still smaller cut-offs in solar distance for meteoroid production in section 4.

The orbit of 109P/Swift-Tuttle has been determined with accuracy backward nearly 2000 years. Marsden *et al.* (1993) and Yau *et al.* (1994) have used observations from the 1992 perihelion passage along with older observations extending back to 69 BC to reverse integrate the equations of motion of the comet. Their independently derived results have a high level of agreement. We use these orbits as the initial seed orbits for all models, noting that the slight difference between the ephemera is much smaller than other uncertainties in our adopted models.

The shape (more precisely the cross-sectional area to mass ratio) of the meteoroids comes into play not only during the ejection process but also in the particles' subsequent evolution under radiation forces. Gustafson (1989) has noted the large variation in ejection velocity predicted solely on the basis of modest variations in the shape factor for meteoroids. Similar work by Nakamura *et al.* (1994) supports the notion that shapes other than the idealized sphere would tend to have higher ejection velocities. We discuss our attempts to account for this effect in Sect. 4.3. The effect of shape on radiation pressure is significant only for the smallest of meteoroids considered here and is discussed further in section 3.

Past attempts to model meteoroid streams (cf. Williams, 1993 for a review) have relied almost entirely on the Whipple model and the numerical relation he determined assuming gas drag lifts a spherical meteoroid away from the sunward side of the nucleus, namely

$$V_{eject} = 8.03 r^{-1.125} \rho^{-\frac{1}{3}} R_c^{\frac{1}{2}} m^{-\frac{1}{6}} \tag{4.1}$$

where R_c is the radius of the cometary nucleus in km, ρ the bulk density of the meteoroid in g/cm^3, m the mass of the meteoroid in grams, n the fraction of incident solar radiation used in sublimation, r is heliocentric distance in A.U. and V_{eject} is the final grain ejection velocity relative to the nucleus in m/s. A typical value of these parameters (R_c=5 km, ρ=0.800 g cm^{-3}, n=1, and m=0.1g) results in a V_{eject} of 36 ms^{-1} at 1 A.U. Note that we have explicitly ignored the gravitational attraction of the nucleus in Eqn 4.1.

Indeed, the Whipple ejection formula provides the starting point for much of the modelling we perform. The shortcomings of the Whipple model, namely the assumption of blackbody limited nucleus temperature (instead of sublimation temperature limited) and the neglect of the adiabatic expansion of the gas have been corrected by (among others) Jones (1995) and we use his revised Whipple formula

$$V_{eject} = 10.2 r^{-1.038} \rho^{-\frac{1}{3}} R_c^{\frac{1}{2}} m^{-\frac{1}{6}} \tag{4.2}$$

for our basic model. In particular, the Whipple formulation ignores the role of isolated jets of activity, which is taken into account in the Jones' model. Despite the modifications, the Jones' equation is very close to that of the original Whipple model. We examine the effects of changes in ejection cone angle (the angle between the solar-direction and velocity vector) to the final results in Sect. 4.4.

Of the parameters in the Jones' formula, the radius of the nucleus is most certain in the case of 109P/Swift-Tuttle. From visible observations of the bare nucleus, Boehnhardt *et al.* (1996) conclude that the nucleus has a radius of 11.2±0.3 km, while O'Ceallaigh *et al.* (1995) have found that the nuclear radius is 11.8±0.2 km using similar observations. Fomenkova *et al.* (1995) derived a radius of 15±3 km from observations in the IR. These extremely large radius estimates are consistent with the apparent lack of non-gravitational forces needed to explain Swift-Tuttle's motion over the last two millennia (Yau *et al.*, 1994). We adopt a radius of 10 km throughout and note that this is almost twice the mean nuclear radius of Halley.

Theoretical models are no better than the assumptions on which they are based and if we ignore for the moment the details of the models we see that they agree on many of the parameters which govern the speed of ejection of the meteoroids. Of particular interest to us is the variation of the ejection speed with the Sun-comet distance. Both the Whipple-derived theories and most other models predict that the variation should be of the form

$$V \propto r^n \tag{4.3}$$

For the Whipple-like theories n is close to -1 while from observations of coma ejections/halo expansions (cf. Whipple, 1980; Combi, 1989), n is close to -0.5. While there can be much discussion on theoretical grounds as to what is the most appropriate value to adopt in practice, at this stage of the process we choose to investigate both possibilities and to make the final choice on the basis of which better describe the observed activity of the stream.

Another shortcoming of the Whipple approach is its assumption that all sublimation is confined to the nucleus surface and is the sole source for gas in the coma. Data gathered during the Halley fly-bys in particular have suggested that sublimation occurs throughout the coma as active grains continue evaporating and releasing H_2O. This contention is supported by the observation that cometary coma gas distributions tend to be spherical despite the presence of jets of activity, that the near-nucleus brightness of the coma drops off slower than l^{-1} (where l is the distance from the surface of the nucleus) as expected for surface production away from the surface and that the terminal dust grain velocity inferred from cometary tail observations shows a weak mass dependence, suggesting that fragmentation of large grains far from the nucleus might be the source for many of the smaller grains. This concept of "distributed" production in the coma is not new but Crifo (1995) has recently incorporated the concept of distributed production into a general physicochemical model of the inner coma along with detailed numerical results of the resulting effects on the terminal dust velocity as a function of mass. He finds that dust ejection velocities for a given mass are broad distributions which tend to have velocity peaks lower than the "classic" surface production models as compared to the single valued velocities derived from the Whipple model. Steel (1994) has emphasized the need to incorporate this effect in the cometary coma into meteor stream modelling, but to date this has not been done.

4.2.2 Constraints from meteor data

Recently, Harris and Hughes (1995) examined photographic meteor data in an attempt to use such information to constrain the cometary ejection process for the Perseids. In particular, their work (as well as that of Williams (1996)) has concentrated on the distribution of semi-major axes of stream meteoroids. These authors suggest that, if no substantial planetary perturbations affect a meteoroid, it is possible to use the true semi-major axis of the particle along with assumed distributions of ejection directions and locations along the cometary orbit to constrain the ejection velocity of the meteoroids. Indeed, Harris and Hughes (1995) suggest that there is no sensible variation in the semi-major axis distribution with meteoroid mass and conclude that all meteoroids reach essentially the same final velocity independent of mass. By comparing the observed distributions of semi-major axes to trial distributions, they suggest that this velocity is close to the final mean gas velocity, about 0.6 km/s for Swift-Tuttle at perihelion.

In using the photographic data of the stream compiled from more than a half dozen different surveys, the effects of measurement errors have not been discussed in detail by either Harris and Hughes (1995) or Williams (1996).

These data consist of Perseid orbits derived from the photographic databases of the 1 A.U. Meteor Data Centre (Lindblad, 1991). To find a value for a (semi-major axis) from photographic observations the original heliocentric velocity must be determined. In measuring the atmospheric velocity, however, a number of possible errors may be encountered, among them:

- The measured velocity in the atmosphere must be corrected for deceleration of the meteoroid over the course of the length of the trail, but this can only be done in an approximate manner. Older observations have used the classic $dv/dt=a+bt+ce^{kt}$ empirical velocity correction (Jacchia and Whipple, 1961) whose validity is questionable and which yields results different from modern applications of methods to account for deceleration such as the gross-fragmentation model of Ceplecha *et al.* (1993).
- For short trails, the number of measured points may be limited and the resulting velocity uncertain. This is particularly a problem with Perseids, which tend to have very short-lived trails in the atmosphere.
- Wake, fragmentation and flares along the trajectory may make measurement of the trail breaks difficult.
- Instrumental effects, particularly related to the frequency of the shutter, can lead to systematic errors. Such effects have recently been found (and removed) from the photographic observations of the Lost City fireball (Ceplecha, 1996)

The same photographic databases used by the previous authors have been examined in detail by Kresakova (1974) and Porubcan (1977) in relation to the Perseids. They have shown that among the dozen major photographic surveys, intersurvey deviations of the rms intrasurvey variation in the measured heliocentric velocity for Perseid meteoroids (which is approximately 41 km/s at 1 A.U.) vary from 0.3 km/s to more than 2.0 km/s, with the majority of surveys greater than 1 km/s. At 1 A.U. the measured heliocentric velocity is related to the semi-major axis via

$$V_h{}^2 = GM(2 - \frac{1}{a}) \tag{4.4}$$

where G is the universal gravitational constant, M the mass of the sun and V_h is the heliocentric velocity in terms of the circular velocity at 1 A.U. Fractional errors in velocity translate into very large errors in a, especially for large values of a (such as the Perseid stream orbit). More precisely if a >>1 then from

$$\frac{da}{a} = 4a\left[\frac{dV_h}{V_h}\right] \tag{4.5}$$

which implies that the smallest rms intrasurvey deviations in V_h for the Perseids corresponds to error dispersions in a of nearly 100%. The bulk of the data have much higher errors, which would be expected to push a beyond the hyperbolic limit. In fact, nearly 1/3 of all available Perseid orbits are at or beyond the parabolic limit, though none of these are seriously considered hyperbolic.

The conclusion for the Perseids is that the distribution of semi-major axis observed by even the most sensitive techniques currently available still produces no useful information concerning the initial conditions of ejection of the stream meteoroids. Kresak (1992) has recently reached a similar conclusion.

While semi-major axis distributions are prone to large errors masking original ejection velocity information for the Perseids, geocentric radiant distributions and flux information for the stream do not suffer as greatly. Indeed, such information provides the basis for the interpretation and validation of the results of our modelling and help to discriminate the most probable initial conditions for the ejection of Perseid meteoroids. These data are presented in Sect. 4.4 along with a discussion of the model results.

4.3 The Initial Ejection Models

4.3.1 Overview

From the forgoing discussion, it is clear that an ejection model of the classic-Whipple type alone does not cover the many possible important variations in ejection conditions, which current observational data and theoretical modelling suggest are possible. As the differences in the final meteoroid distributions may be sensitive to the initial model choices, it is desirable to use several different ejection schemes and compare the final results. The resulting differences will determine which models are best able to fit the available Perseid observations assuming the intermodel differences are great enough to distinguish the outcomes.

After reviewing the available information on the cometary ejection process as summarized in Sect 4.2, we have decided to use four major models of ejection of meteoroids from 109P/Swift-Tuttle.

The first model uses the results of Crifo's (1995) coma modelling for distributed production in the coma. His result for the average terminal velocity of the dust (appropriate for grains from 10 cm > s > 10^{-4} cm) for olivine grains as a function of grain radius, *s*, can be expressed empirically as:

$$Log_{10}(V_{eject}) = -2.143 - 0.605 Log_{10} s \tag{4.6}$$

and we assume the production varies with heliocentric distance as $r^{-0.5}$. The result is scaled from his simulation work (which was designed for Halley to compare the final results with Giotto measurements) to that appropriate for 109P/Swift-Tuttle assuming the same fractional area on both comets was active. This value for the average velocity (V_{eject}) from Eqn 4.6 is then used along with Crifo's velocity distribution for the differential flux as a function of velocity for a mass of 10^{-2} g, which has an empirical form of:

$$P(V-Veject)=\frac{1}{e^{3.7}}\exp\left[\frac{3.7-10.26(V-Veject)+4.12(V-Veject)^2}{1-1.03(V-Veject)+0.296(V-Veject)^2}\right] \tag{4.7}$$

where *P(V-Veject)* is the relative probability of finding a grain with ejection velocity V. This is model 1.

The second model is the Jones modification to the original Whipple formula with the exception that the solar distance dependence on ejection velocity is taken to be $r^{-0.5}$. We call this variant model 2.

As the Whipple model has been used by almost all previous workers in modelling streams it seems appropriate for comparison of our final results to past results to include this model. The slight modification to the Whipple model by Jones is used and we call this model 3 throughout. It is similar to 2 except that the heliocentric velocity dependence is $r^{-1.038}$.

The fourth and final model uses the same ejection velocity formulation as model 3, with the exception that it is not a single-valued function for a given choice of input parameters. Instead, we use a parabolic distribution centred about the nominal Jones velocity in an attempt to account for the different ejection velocities for a given mass due to the differing shape factors. Since we have no numerical constraints a priori regarding grain shapes, we use this parabolic distribution in an attempt to account for this variation. This is model 4.

For each model, the absolute value for the grain ejection velocity will vary as a function of the chosen meteoroid density. Estimates for cometary nucleus densities vary widely, with evidence from Halley suggesting values in the ~100 kg m^{-3} range (Rickman, 1986) or lower, while Sagdeev *et al.* (1987) estimate this value to be closer to ~600 kg m^{-3}. However, the nucleus density may have little relationship to the density of smaller grains. Indeed, Ceplecha (1988) and Verniani (1973) have analyzed fireball and radio meteor sized bodies (10^{5}-10^{-4} g) and find bulk densities near 800 kg m^{-3}. In contrast, Babadzhanov (1993) finds densities closer to ~4000 kg m^{-3} from photographic meteor data and the application of a fragmentation model to the observed data. These wide ranges for the possible densities of Perseid meteoroids have led us to adopt three distinct densities we use for all models; namely 100 kgm^{-3}, 800 kgm^{-3}, and 4000 kgm^{-3}, which we enumerate as 1,2, and 3 model variants. Thus the distributed production model with meteoroids of density 100 kgm^{-3}, 800 kgm^{-3}, and 4000 kgm^{-3} are referred to as models 11, 12 and 13 respectively. The ejection velocity formula for each model is given in Table 4.1 and sample distributions for ejection velocities as a function of heliocentric distance are shown in Fig. 4.1 for Perseid meteoroids of mass 10^{-2} g.

We have taken the meteoroid mass to be the independent variable and plot all results in terms of initial ejection mass. In total we have 12 distinct model variants and for each we eject 10 000 test meteoroids at differing masses from 10^{-5}-10 g for each perihelion passage of 109P/Swift-Tuttle. We have used 61 mass categories over this mass range for the 1862 and 1737 passages of the comet for each model variant - each mass category is 0.1 greater in Log(M(g)) space than the previous category. This implies a total of 610 000 test meteoroids are ejected for each model variant, totalling 7.32×10^6 particles for each passage (1862 and 1737). For passages from 59-1610 A.D. only 7 mass categories are used over the full mass range due to computational limitations, each 10 times greater than the previous (1.0 in Log (M(g)) space)) totalling 8.4×10^5 meteoroids per perihelion passage. These choices for mass, coupled with the three chosen values for densities imply a range of β in our simulations of $10^{-5}<\beta<10^{-2}$.

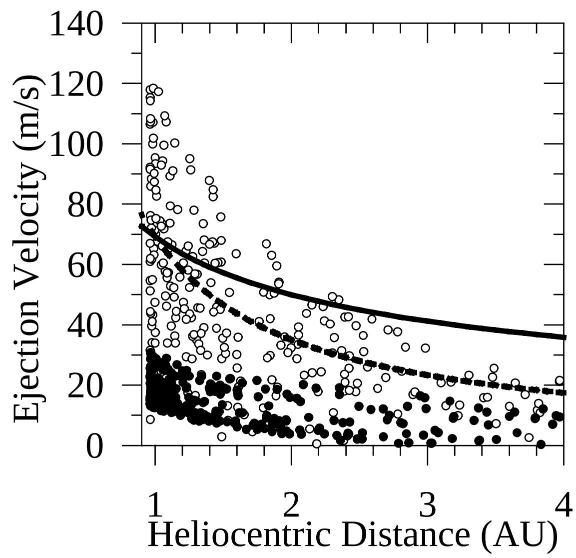

Fig 4.1: Sample ejection velocity distribution for Perseids ejected in 1862 of mass 0.01g and density 800 kg m^{-3} (β=5×10^{-4}) as a function of heliocentric ejection distance. Model 1 meteoroids are shown as filled circles, model 2 as a solid line, model 3 as a dotted line and model 4 as open circles.

For each model variant, the same basic Monte Carlo approach is taken to determine the point of ejection and ejection velocity/direction. As described in Sect. 4.2, the point along the orbit of 109P/Swift-Tuttle where ejection occurs is chosen randomly from within the true anomaly range from 233°<ν<127 °or r<4 A.U. After this ejection point is determined, the appropriate ejection speed is then found, depending on the model variant, using one of the formulae given in Table 4.1. The direction of ejection is confined to the sunward side of the comet and randomly chosen while the final ejection magnitude is calculated according to each model formula. The resulting cometocentric velocity is added to the cometary velocity at the ejection location to derive the initial orbit. This process is repeated for all 10 000 meteoroids for a particular run and this file is then used as the input to the numerical integrator.

Table 4.1 : Formula for determining the Ejection velocity of a meteoroid of mass *m* from Swift-Tuttle for each model variant.

Model #	Name	Ejection Formula
1	Crifo Distributed Production	$Log_{10}(V_{eject}) = -2.143 - 0.605 Log_{10}(radius) - 0.5 Log_{10} r$ $P(V - Veject) = \frac{1}{e^{3.7}} \exp\left[\frac{3.7 - 10.26(V - Veject) + 4.12(V - Veject)^2}{1 - 1.03(V - Veject) + 0.296(V - Veject)^2}\right]$
2	Jones Ejection Distribution with Modified Heliocentric Velocity Dependence	$V_{eject} = 10.2\, r^{-0.5} \rho^{-\frac{1}{3}} R_c^{\frac{1}{2}} m^{-\frac{1}{6}}$ P(V-Veject) = 1 for V=Veject and 0 if V≠ Veject
3	Jones Ejection Distribution	$V_{eject} = 10.2\, r^{-1.038} \rho^{-\frac{1}{3}} R_c^{\frac{1}{2}} m^{-\frac{1}{6}}$ P(V-Veject) = 1 for V=Veject and 0 if V≠ Veject
4	Jones ejection distribution with parabolic probability distribution	$V_{eject} = 10.2\, r^{-1.038} \rho^{-\frac{1}{3}} R_c^{\frac{1}{2}} m^{-\frac{1}{6}}$ $P(V - Veject) = 1 - (\frac{V}{Veject} - 1)^2$ for 0<V<2Veject and 0 outside

4.3.2 The Numerical Integrator

The basic form of the numerical integrator uses an RK4 architecture with variable step-size. Jones (1985) described an early version of this integrator where more details can be found. This integrator has been specifically designed for integrating large numbers of bodies as quickly as possible over (relatively) short solar system times. Whereas typical integrators used in solar system work such as RADAU (Everhart, 1985) or SWIFT (Levison and Duncan, 1994) are designed for high precision and long-periods of integration, we are concerned with maintaining only modest precision and concentrating instead on particle throughput.

To this end, the integrator uses a simple RK4 numerical integration scheme adapted from Press *et al.* (1986). The basic step-size was chosen initially based on numerical experiments offsetting speed and accuracy - a typical value being 0.01 years. For an orbit as eccentric as 109P/Swift-Tuttle, variable step-size routines we tested suggested that the large number of steps near perihelion did not increase the overall orbital accuracy (our primary interest) and that the resulting numerical round-off errors and loss of speed were significant. Jones (1985) found that an empirical formula of the form $h=h_0r^p$ where r is the distance to the closest major body in the integration and p is chosen empirically provides an acceptable compromise between speed and numerical accuracy. For orbits as elliptical as 109P/Swift-Tuttle a value of p=1.5 is close to optimum in the product of integration time and final total accumulated error and we use this throughout. Other integration schemes are available which are superior in speed and produce somewhat more precise results. For our purposes, however, the RK4 integrator is entirely adequate and has been tested against output from SWIFT and RA.DAU and found to show no variations of significance within our range of adopted bin sizes in parameter space.

To further speed up integrations, the $(n-1)^2$ computations normally found in n-body calculations (and general features of other solar system integrators) were removed entirely by generating pre-defined planetary position tables in memory. These tables were derived from the DE404 JPL planetary ephemeris and are stored in computer memory with planetary positions interpolated via cubic splines to accuracies (relative to the original DE404 ephemeris) no worse than 100 km for the positions of the major planets over the last 2000 years, with average errors nearly one order of magnitude better than this value.

All numerical computations are performed taking into account planetary perturbations, barycentric corrections, radiation pressure and the Poynting-Robertson effect (cf. Chapter 2 for a basic description and Burns *et al.*, (1979) for a detailed description of the latter two forces). The barycentric corrections are significant for orbits as large and elliptical as 109P/Swift-Tuttle (cf. Chambers, 1995) and necessitated an upper limit of between 0.2 - 0.4 years in the largest step-size, independent of distance to the nearest perturbing body.

The above integrations required approximately four months of continuous computation on five Pentium PC's.

4.4. Results

4.4.1 Previous Perihelion Passage (1862)

We begin by examining the meteoroid distribution at the present epoch due to Perseids ejected in 1862. Some general comments concerning the overall evolution of the modelled meteoroids from 1862 to the present are in order. It was found that models 2 and 3 show virtually identical outcomes both in terms of flux as a function of time and solar longitude of maximum in any given year, locations of radiants, stream dispersion etc. The choice of $r^{-0.5}$ or r^{-1} dependence on the ejection velocity was found to be the most insensitive variation among models from 1862 and always resulted in very similar final distributions. As the majority of the meteoroids are ejected near perihelion (as required by the condition of random distribution in true anomaly), the small number of more distant ($r>2$ A.U.) ejections do not make a strong contribution to the overall activity of the stream presently observed in the context of our modelling. The only noticeable difference between the more distantly ejected population ($2<r<4$ A.U.) and meteoroids ejected near perihelion is a larger spread in nodal longitudes for the former which becomes particularly evident at small masses.

Fig 4.2 shows a temporal plot of the distribution by mass of test meteoroids having nodes within 0.005 A.U. of the Earth's orbit from model 32. We use 0.005 A.U. as our sieving distance and hereafter refer to all such meteoroids as Earth intersecting. Smaller sieving distances were used, but found to be inconsequential for the Perseids in overall terms. There is an obvious periodicity in the figure apparent in all model variants of ejecta from 1862. Fig 4.3 shows a plot of nodal distance versus time for model 42 meteoroids of mass 10^{-2} g, demonstrating that the reason for the periodicity is an impulsive change in the mean nodal distance of shower meteoroids inwards every 12 and 30 years. This effect is the result of distant direct perturbations on the stream by Jupiter and Saturn and is developed in more detail in Sect 4.5.

In general, all models show that the most recent activity associated with the 1862 ejecta is concentrated from 1991-1994 with a peak in 1993. It is clear from Fig 4.3 that meteoroids not perturbed by planetary perturbations after ejection in 1862 have nodes outside Earth's orbit, a result that holds for all models and all masses. In rare cases, smaller meteoroids (generally of higher density) ejected with high velocities can reach within 0.005 A.U. outside of Earth's orbit and be "accepted" as visible at Earth in years well away from the inward nodal shifts due to planetary perturbations, but this number is very small. Some activity is also apparent near 1980 and near 2010 at lower levels.

For activity in any year from 1992-1994, the distribution of nodes for all models is strongly concentrated in the region from 139.3°-139.6°, with maximum in the region 139.42°-139.5°. This result changes with cone angle in such a way that smaller cone angles tend to concentrate the peak into a smaller range of solar longitude centred about the node of the comet (139.44°) as would be expected. The particle distribution in these years from the 1862 ejection is also heavily skewed toward the largest (lowest ejection velocity least radiation pressure affected) masses.

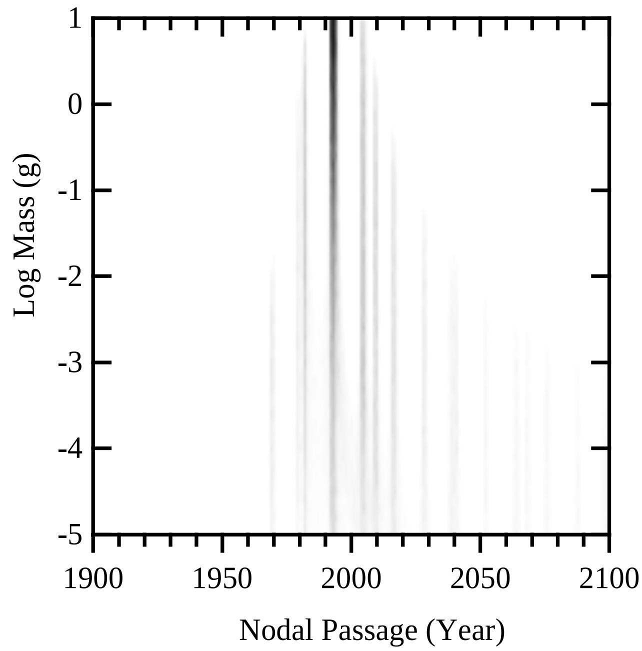

Fig 4.2: Activity at the present epoch from ejecta released in 1862 for model 32. The gridding is 1 year bins for the nodal passage time and 0.1 in log M. The greyscale has a dynamic range from 0 to 700 for this choice of binning intervals.

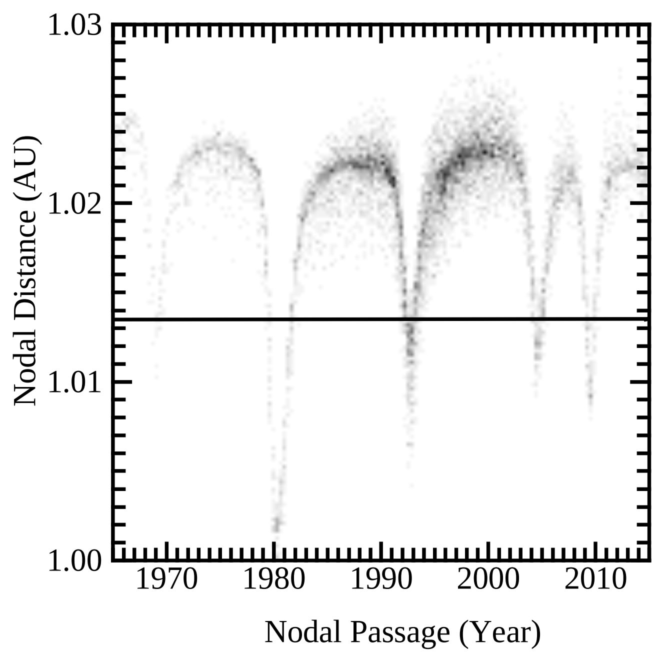

Fig 4.3: Nodal distance versus nodal passage time for meteoroids ejected in 1862 from model 42 of mass 0.01 g. The Earth's distance at the time of Perseid maximum is 1.01355 A.U. and is shown by the horizontal line. The greyscale has a dynamic range from 0 to 19 for this binning. The gridding is to a resolution of 0.0002 A.U.

The radiant size is determined entirely by the distribution of initial ejection velocities; for the models used here, the 1862 radiant rms diameter is ~0.1 degrees. The location of the radiant varies from year to year by a small amount (about 0.3 degrees in declination and 0.2 degrees in RA) due to differential planetary perturbations.

4.4.2 Recent Ejections (2000 years).

Results of ejections from 109P/Swift-Tuttle at each perihelion passage from 59 A.D. to 1610 A.D. were carried out at 7 discrete mass intervals separated by one order of magnitude in mass in the range 10 g-10^{-5} g. For completeness, the same mass categories were extracted from the more extensive runs from 1737 and 1862.

The final distributions of meteoroids at the present epoch reveal that the difference in closest approach between the comet and Earth at the epoch of ejection is a strong determinant of subsequent activity.

Fig 4.4 shows a plot of the minimum approach distance between the osculating orbit for 109P/Swift-Tuttle at the epoch of each perihelion passage (listed as years in the abscissa) and the Earth. The dashed line shows the total number of meteoroids from all models ejected from each passage, which still have nodes within 0.005 A.U. of Earth at the nodal passage closest to the 1992 perihelion date. There is no significant correlation between the age of ejection over this time interval and the fraction of all ejected meteoroids currently in earth-intersecting orbits. This finding suggests that the Earth-comet orbit distance at the time of ejection, rather than planetary perturbations, control the large scale delivery of Perseid meteoroids on this time scale. It is for this reason that material ejected in 1737 and 1610, though quite young, is expected to be less prolific on average at present than ejecta from 1479. Indeed, it is found that for the years from 1995-1997, for example, the material from 1479 is the dominant Perseid population observed at the Earth for the outburst portion of the stream. A similar trend is seen for each model, further indicating that neither the assumed particle density or ejection velocity plays a dominant role in the subsequent encounter conditions with the Earth.

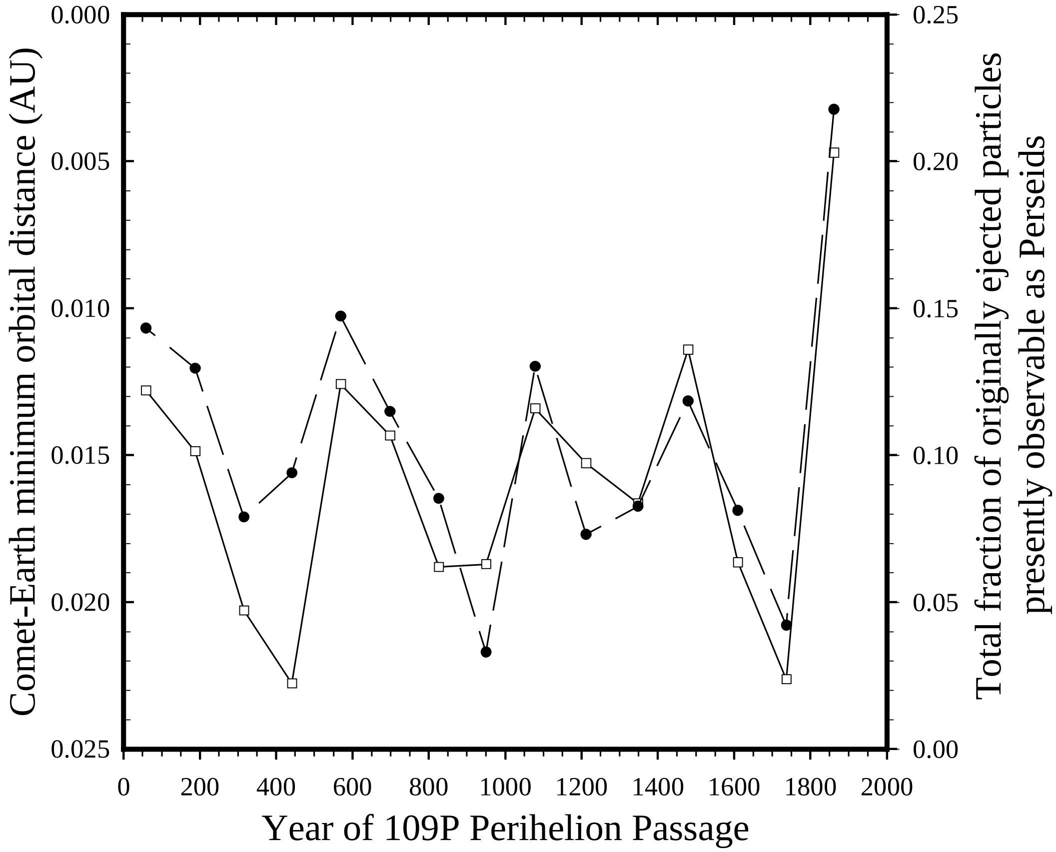

Fig 4.4: The minimum comet-Earth distance for 109P/Swift-Tuttle (open squares) and the fraction of Perseid meteoroids summed over all models and all masses which currently have descending nodal points within 0.005 A.U. of the earth from each perihelion ejection (solid circles).

The total number of Earth intersecting Perseids as a function of time at the present epoch summed over all ejecta for meteoroids capable of producing visual meteors (>10^{-3} g and larger) over the last 2000 years is shown in Fig. 4.5 for three representative models. The general form of the activity is similar for all 12 model variants - namely a 12 and 30 year periodicity reaching peak strength near 1992-1993. For each ejection model, higher meteoroid densities (smaller β's) yield more Earth intersecting meteoroids, a result of the general trend toward larger nodal distances as radiation pressure increases at larger β (see Sect. 4.5). The year of ejection associated with the most numerous population of meteoroids varies significantly from year to year in the current epoch; as a result we expect that the position of peak activity in the stream for the outburst component will similarly vary.

The rms angular width of the radiant as a function of time is shown in Fig. 4.6. Here we have plotted the rms spread in the distribution of individual geocentric radiant points calculated from each Earth-intersecting visual-sized Perseid and added the distributions in a cumulative manner. Hence, the value at 2000 years is the angular spread in the total radiant area from all 15 perihelion ejections from 59 A.D.-1862. Note that the positions of the radiants from any one ejection vary in RA and DEC due to planetary perturbations; thus the rms spread in this cumulative plot is greater than the individual radiant spreads from each individual ejection. The initial size of the radiants and early evolution of the size of the radiant area are controlled by the ejection velocity, with higher average velocities having larger initial dispersions, but within 500 years (roughly four passages) the absolute levels of spread vary inversely with the density of the meteoroids for all models. This suggests that in the longer term, the absolute level of rms spread is controlled (either directly or indirectly) by radiation pressure and to a lesser degree by the initial ejection velocity. However, the slope of the radiant dispersion is constant and similar for all models, showing that planetary perturbations and initial ejection comet-Earth geometry are the "drivers" of the actual shape of the radiant.

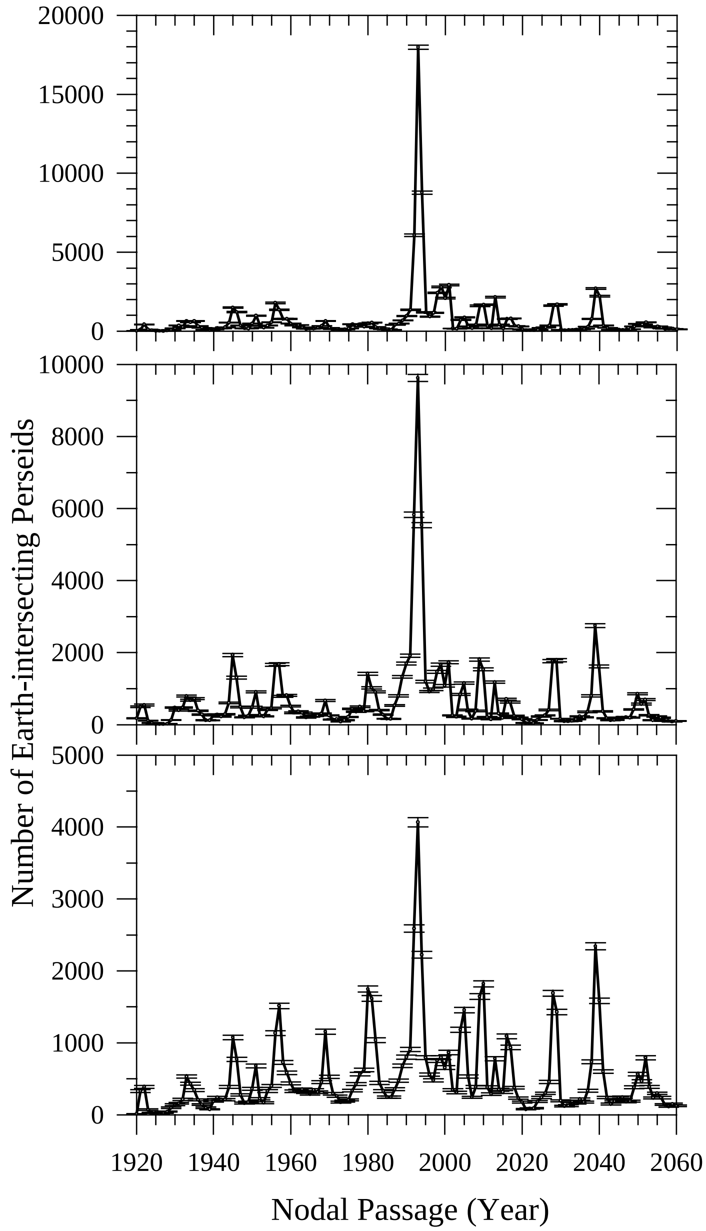

Fig 4.5 : The total number of visual class (>10^{-3} g) Earth intersecting Perseid meteoroids versus their nodal passage time summed from all ejections from 59-1862 A.D. for models 12 (top), 33 (middle) and 41 (bottom). Activity is summed into yearly bins and the error bars represent the poisson error margins ($n^{1/2}$).

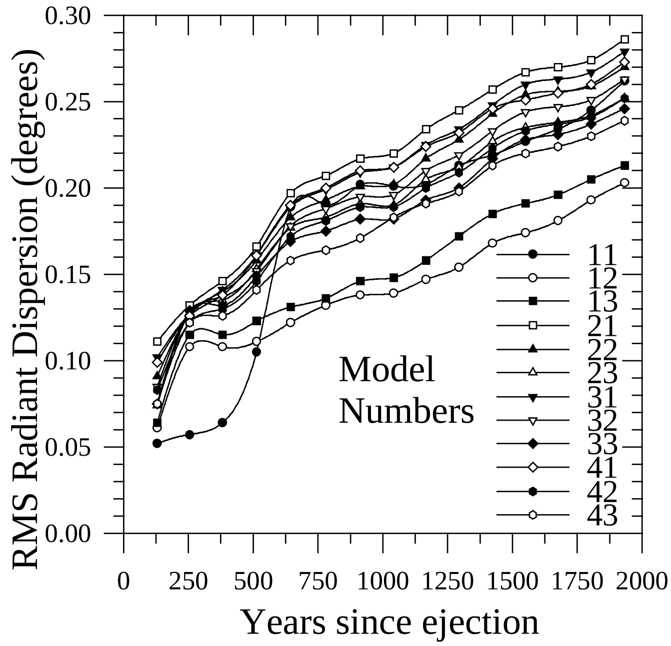

Fig 4.6: Root-mean squared (rms) spread in radiant size for all models as cumulative distributions observed at present from ejections between 59-1962 A.D. The time in years (abscissa) refers to years before present (i.e. year 0 is 2000 A.D.).

A regression fit to the radiant spread from 500-2000 years yields an annual change of 6.5×10^{-5} degrees/annum. The particularly small values for model 1 are a direct consequence of the extremely low ejection velocities associated with the extended production model. The correctness of the above conclusions can be evaluated through consideration of the very low initial dispersion for this model due to the extremely low ejection velocities for larger particles in model 11 (or equivalently for lower density particles of the same mass - the opposite to the dependence from the other models - see Eqn 4.6) and its sudden increase relative to models 12 and 13 after four to five revolutions as the full effects of radiation pressure expand the radiant.

The scatter in the rms spread at any one time between all models is of order 0.1 degrees over this 2000 year period. The location of the radiant after the full 2000 years of accumulated ejections (i.e. now) over the mass range 0.1-10 g (photographic) is α=46.09±0.02 and δ=57.66±0.02 (J2000.0). The variation in α throughout this time is very linear and well represented by α=45.88+1.128145$\times$10^{-4}Y where Y is the year of the last included ejection figured backwards in time in the summation, referenced to an origin at 2000 A.D.. The declination shows much more scatter during the last 2000 years as it depends more on planetary perturbations than α (which is more closely linked to the progression of the node). The variation is approximately represented by δ=57.67+10^{-5}Y. All radiant measures are referenced to J2000 and λ_o=139.7° (139.0° in B1950.0).

The locations and strength of the observed visual peak associated with the outburst component of the stream derived from Chapter 3 and from Rendtel and Arlt (1996) are shown in Fig 4.7, together with the model predictions for the same quantities. The locations of the visual peaks in outburst activity and their shape were found by taking the average Perseid ZHR profile from Chapter 3 over the period 1988-1994 and subtracting this profile from each year's activity after scaling for differences in peak activity between the average profile and each yearly profile's main (or normal) maximum ZHR value. It was found that the mean curve of Perseid ZHR activity from 1988-1994 in the interval from 139°<λ_o<140.1° is approximated by:

$$ZHR = 1.84110984\times10^{8} - 3.95803796\times10^{6}\lambda_O + 28363\lambda_O^{2} - 67.67\lambda_O^{3} \quad (4.8)$$

From Fig 4.7, it is clear that the predicted and model times of peak are generally in good agreement, with the exceptions of the 1993 and 1994 peak locations, where model values are 1-2 hours earlier than observed. The overall trend of observed changes in peak location and the model locations are consistent, reflecting the dominance of older ejecta before 1992 and after 1994 (see Table 4.2). Note that the ZHR of the outburst peak in 1988 was found to be of negligible magnitude after subtraction of the mean scaled background, drawing into doubt the reality of the feature in 1988. We thus omit it from further analysis. The move in the time of the peak away from the current nodal longitude (139.44°) of Swift-Tuttle reflects the fact that 109P's nodal longitude has been higher than its present value for most passages over the last 1000 years and hence older ejecta are now well ahead of the comet's nodal longitude. This ejection geometry implies that ejecta from as recently as 1348 can be found as late at nearly 139.8° at the present epoch, all other ejections over the last 2000 years peaking earlier.

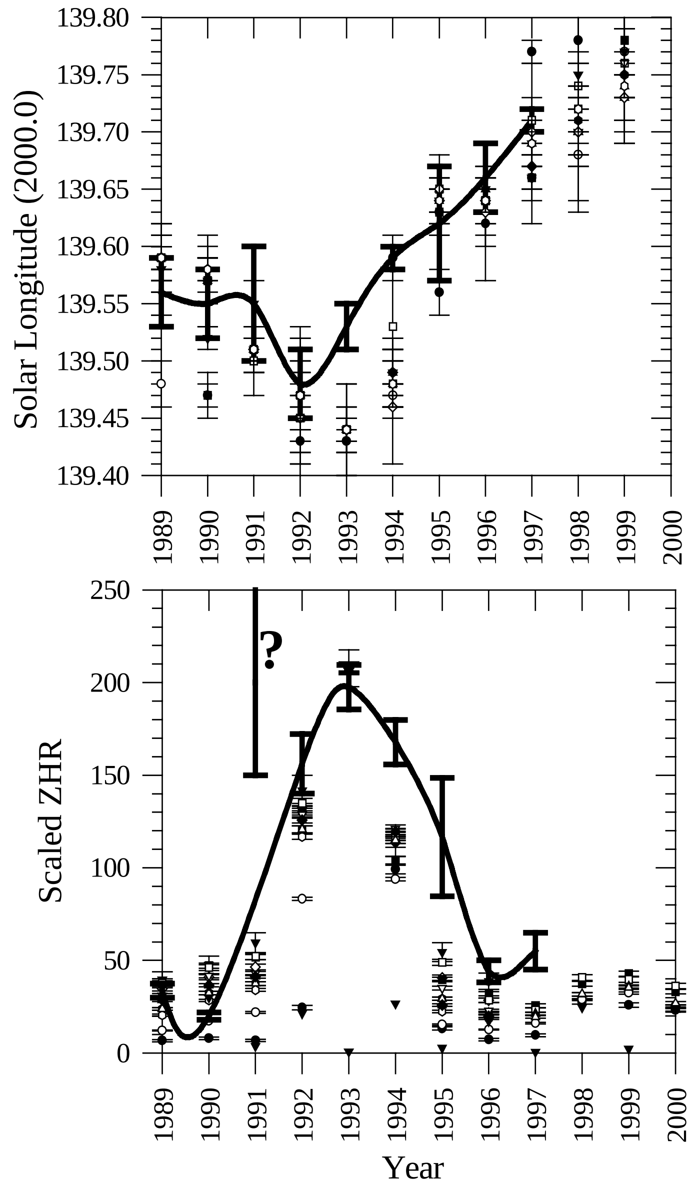

Fig 4.7: Observed locations (J2000) of the outburst peak for the Perseids (bold solid line) together with model predictions of peak locations (top). The scaled ZHR for the observed outburst peaks from 1989 to 1997 and individual model predictions are also given (bottom). Symbols for each model variant for both plots are the same as used in Fig 4.6. Observational data are from Chapter 3. The question mark next to the 1991 observed ZHR value reflects the high uncertainty of this datum (see discussion of this point in the previous chapter). The solid curve for the bottom graph is formed without using 1991.

The cumulative activity for visual-sized meteoroids is shown in Fig 4.8 for two representative models as a function of solar longitude. All meteoroids ejected over the last 2000 years currently have nodal longitudes greater than 139° and the profile from just these 15 ejections already shows remarkable similarity to the shape of the core Perseid activity found from visual observations, the asymmetry of both being particularly notable.

The relative change in the strength of the peaks is reproduced, though the peak observed ZHR in 1991 (which has large error margins) does not fit the trend well, the model underestimating its strength. A similar, though less substantial effect, is also seen in 1994 and 1995 suggesting that for the strongest years the model tends to underestimate peak ZHR activity.

Table 4.2 summarizes the age breakdown of the recent Perseid activity (all of the outburst peak and part of the core activity of the stream) in terms of the summation of all ejections (59A.D. - 2000) for each of the years 1988-1996 for visual class meteoroids. The total number of Earth intersecting test meteoroids as well as the fraction of this total contributed by the three most significant ejections is also shown. It can be seen that the activity for all models peaks in 1992-1993 and that the makeup of the ejecta observed as the outburst component of the Perseids changes dramatically from year to year. In 1988-1990, ejections from 1610 and 1737 are predominant and account for the majority of the activity, while in 1991 material from 1862 and 1610 is found in roughly equal proportions. The 1991-1994 outburst maxima are composed primarily of material from 1862 and to a lesser degree 1610. Note that even in these years, the fraction of all meteoroids of recent origin (last 2000 years) is still greater from all earlier passages than from 1862 alone. In 1995 and 1996 the origin of the outburst material changes again, with 1479 dominant and 1079 and 1862 making contributions. These age breakdowns and total numbers of accepted particles refer to the integrated flux (fluence) of meteoroids over the full activity interval of the shower while the higher fluxes are most likely to be associated with younger material more concentrated in solar longitude.

	11		12		13		21		22		23	
1988	**95**		**472**		**748**		**589**		**688**		**716**	
	441	**0.24**	**1737**	**0.71**	**1737**	**0.56**	**1610**	**0.26**	**1610**	**0.39**	**1610**	**0.48**
	698	**.24**	**1610**	**0.15**	**1610**	**0.32**	**1862**	**0.16**	**1737**	**0.20**	**1737**	**0.31**
	59	**0.13**	**441**	**0.04**	**826**	**0.03**	**1479**	**0.15**	**1479**	**0.10**	**826**	**0.05**
1989	**122 (1.73)**		**691 (1.14)**		**1178(.93)**		**659 (.45)**		**961(.31)**		**1277(.27)**	
	698	**0.21**	**1737**	**0.52**	**1610**	**0.49**	**1610**	**0.32**	**1610**	**0.49**	**1610**	**0.55**
	188	**0.19**	**1610**	**0.37**	**1737**	**0.40**	**1862**	**0.17**	**1737**	**0.18**	**1737**	**0.22**
	59	**0.17**	**59**	**0.04**	**59**	**0.03**	**1479**	**0.15**	**1479**	**0.12**	**1479**	**0.08**
1990	**144(.36)**		**965(.97)**		**1681(.95)**		**827(.46)**		**1168(.51)**		**1585(.79)**	
	1348	**0.31**	**1610**	**0.48**	**1610**	**0.54**	**1610**	**0.39**	**1610**	**0.50**	**1610**	**0.58**
	698	**0.14**	**1737**	**0.45**	**1737**	**0.37**	**1862**	**0.20**	**1737**	**0.18**	**1737**	**0.23**
	188	**0.14**	**59**	**0.02**	**1479**	**0.03**	**1737**	**0.10**	**1479**	**0.10**	**1479**	**0.07**
1991	**124(1.51)**		**1351(.97)**		**2127(.88)**		**927(.76)**		**1282(.54)**		**1689(.66)**	
	59	**0.20**	**1862**	**0.37**	**1610**	**0.38**	**1862**	**0.31**	**1610**	**0.34**	**1610**	**0.39**
	569	**0.20**	**1610**	**0.36**	**1862**	**0.36**	**1610**	**0.25**	**1862**	**0.33**	**1862**	**0.33**
	698	**0.11**	**1737**	**0.23**	**1737**	**0.20**	**1737**	**0.10**	**1737**	**0.12**	**1737**	**0.14**
1992	**448(.82)**		**6071(.24)**		**8063(.2)**		**2100(.75)**		**3240(.56)**		**4586(.31)**	
	1862	**0.44**	**1862**	**0.47**	**1862**	**0.43**	**1862**	**0.29**	**1862**	**0.37**	**1862**	**0.41**
	826	**0.18**	**1610**	**0.15**	**1610**	**0.19**	**1610**	**0.15**	**1610**	**0.15**	**1610**	**0.16**
	1079	**0.11**	**1737**	**0.07**	**1479**	**0.08**	**1479**	**0.10**	**1479**	**0.10**	**1479**	**0.08**
1993	**3780(4.96)**		**17971(4.16)**		**20120(3.86)**		**2971(5.56)**		**5142(3.61)**		**7714(3.94)**	
	1862	**0.70**	**1862**	**0.49**	**1862**	**0.42**	**1862**	**0.32**	**1862**	**0.37**	**1862**	**0.39**
	826	**0.04**	**1610**	**0.12**	**1610**	**0.15**	**1610**	**0.17**	**1610**	**0.16**	**1610**	**0.19**
	1079	**0.03**	**1079**	**0.06**	**1079**	**0.07**	**1079**	**0.09**	**1079**	**0.09**	**1079**	**0.11**
1994	**1804(2.86)**		**8773(4.01)**		**9094(5.06)**		**1542(3.54)**		**2578(4.48)**		**4213(5.24)**	
	1079	**0.18**	**1079**	**0.18**	**1862**	**0.21**	**1862**	**0.34**	**1862**	**0.34**	**1862**	**0.28**
	569	**0.16**	**1862**	**0.15**	**1079**	**0.12**	**1479**	**0.19**	**1479**	**0.15**	**1610**	**0.16**
	826	**0.16**	**826**	**0.14**	**826**	**0.12**	**1079**	**0.09**	**1610**	**0.10**	**1479**	**0.14**
1995	**241(0.38)**		**1189(0.16)**		**1482(0.04)**		**779(0.05)**		**983(0.30)**		**1183(0.22)**	
	1079	**0.29**	**1479**	**0.32**	**1479**	**0.49**	**1479**	**0.40**	**1479**	**0.49**	**1479**	**0.56**
	569	**0.14**	**1079**	**0.24**	**1079**	**0.15**	**1862**	**0.27**	**1862**	**0.19**	**1079**	**0.12**
	950	**0.14**	**569**	**0.10**	**698**	**0.09**	**1079**	**0.07**	**1079**	**0.09**	**1862**	**0.08**
1996	**132(3.51)**		**926(1.35)**		**1231(1.19)**		**592(0.82)**		**795(1.20)**		**934(0.90)**	
	1079	**0.20**	**1479**	**0.44**	**1479**	**0.53**	**1479**	**0.48**	**1479**	**0.65**	**1479**	**0.69**
	569	**0.19**	**1079**	**0.20**	**1079**	**0.13**	**1862**	**0.24**	**1862**	**0.10**	**1079**	**0.09**
	950	**0.14**	**569**	**0.09**	**698**	**0.08**	**1079**	**0.05**	**1079**	**0.05**	**698**	**0.04**
Totals	**6890 (16.13)**		**38409 (13.0)**		**45724 (13.11)**		**10986 (12.39)**		**16837 (11.51)**		**23897 (12.33)**	

Table 4.2 : The number of Earth intersecting Perseids by year (from 1988-1994) and by model at the present epoch. Each column title represents the model number. Each row is the model results for the given year. The rows list the total number of particles from a particular model accepted in the given year, followed by a breakdown of the three most numerous ejection epochs represented, with the year of ejection in the left half-column and the fraction of the total number of particles contributed by this ejection in the right.

	31		32		33		41		42		43	
1988	**471**		**655**		**800**		**474**		**626**		**724**	
	1610	0.36	1610	0.41	1610	0.42	1610	0.31	1610	0.38	1737	0.46
	1737	0.23	1737	0.34	1737	0.42	1737	0.26	1737	0.37	1610	0.37
	826	0.08	826	0.05	826	0.04	1862	0.10	826	0.04	826	0.04
1989	**678(.14)**		**948(.45)**		**1327(.39)**		**680(.42)**		**871(.37)**		**1171(.47)**	
	1610	0.44	1610	0.52	1610	0.57	1610	0.40	1610	0.52	1610	0.55
	1737	0.17	1737	0.24	1737	0.29	1737	0.19	1737	0.28	1737	0.31
	1479	0.11	1479	0.08	1479	0.04	1479	0.09	1479	0.07	1479	0.04
1990	**831(.48)**		**1272(.57)**		**1696(.81)**		**802(.70)**		**1185(.72)**		**1628(.88)**	
	1610	0.47	1610	0.54	1610	0.60	1610	0.43	1610	0.54	1610	0.58
	1737	0.18	1737	0.22	1737	0.29	1737	0.19	1737	0.25	1737	0.31
	1862	0.08	1479	0.09	1479	0.06	1862	0.11	1479	0.07	1479	0.05
1991	**940(.41)**		**1308(.77)**		**1916(.77)**		**909(.47)**		**1372(.68)**		**1955(.89)**	
	1862	0.35	1862	0.36	1862	0.39	1862	0.36	1862	0.37	1862	0.39
	1610	0.29	1610	0.34	1610	0.35	1610	0.28	1610	0.33	1610	0.35
	1737	0.11	1737	0.15	1737	0.17	1737	0.11	1737	0.16	1737	0.18
1992	**2428(.51)**		**3926(.32)**		**5827(.22)**		**2588(.32)**		**4410(.18)**		**6692(.21)**	
	1862	0.37	1862	0.40	1862	0.43	1862	0.38	1862	0.43	1862	0.44
	1610	0.12	1610	0.14	1610	0.15	1610	0.13	1610	0.14	1610	0.16
	1479	0.09	1479	0.09	1737	0.09	1479	0.08	1737	0.08	1737	0.09
1993	**3741(5.46)**		**6334(3.93)**		**9623(3.89)**		**4064(3.88)**		**7586(3.97)**		**11896(3.75)**	
	1862	0.38	1862	0.38	1862	0.39	1862	0.37	1862	0.39	1862	0.40
	1610	0.14	1610	0.16	1610	0.17	1610	0.14	1610	0.16	1610	0.15
	1079	0.09	1079	0.10	1079	0.10	1079	0.09	1079	0.10	1079	0.09
1994	**2112(5.44)**		**3693(4.82)**		**5539(4.68)**		**2222(5.24)**		**4196(5.45)**		**6661(5.21)**	
	1862	0.36	1862	0.35	1862	0.32	1862	0.31	1862	0.34	1862	0.32
	1610	0.17	1610	0.18	1610	0.16	1479	0.12	1610	0.14	1610	0.12
	1479	0.10	1479	0.09	1079	0.12	1610	0.12	1079	0.10	1079	0.12
1995	**881(0.23)**		**1071(0.23)**		**1199(0.16)**		**795(0.25)**		**1072(0.03)**		**1283(1.23)**	
	1479	0.36	1479	0.46	1479	0.59	1479	0.41	1479	0.50	1479	0.55
	1862	0.27	1862	0.15	1079	0.12	1862	0.20	1862	0.12	1079	0.14
	1079	0.07	1079	0.07	950	0.05	1079	0.06	1079	0.09	826	0.07
1996	**513(1.71)**		**698(1.20)**		**921(0.87)**		**559(1.50)**		**791(0.94)**		**1072(5.54)**	
	1479	0.50	1479	0.66	1479	0.71	1479	0.51	1479	0.62	1479	0.66
	1862	0.14	1079	0.06	1079	0.08	1862	0.14	1079	0.08	1079	0.10
	1079	0.09	1862	0.06	826	0.04	1079	0.08	826	0.06	826	0.06
Totals	**12595 (14.38)**		**19905 (12.29)**		**28848 (11.79)**		**13093 (12.78)**		**22109 (12.34)**		**33082 (18.18)**	

Table 4.2 (continued): Same as previous page, but covering results from models 31-43. Note that the numbers in parentheses at the bottom represent the difference between the modelled activity profiles and the observed ZHR profiles summed for all years from 1989 - 1994 (see text for more explanation). Additionally, the rms fit for each year is given in parentheses immediately after the total number of test particles for each model.

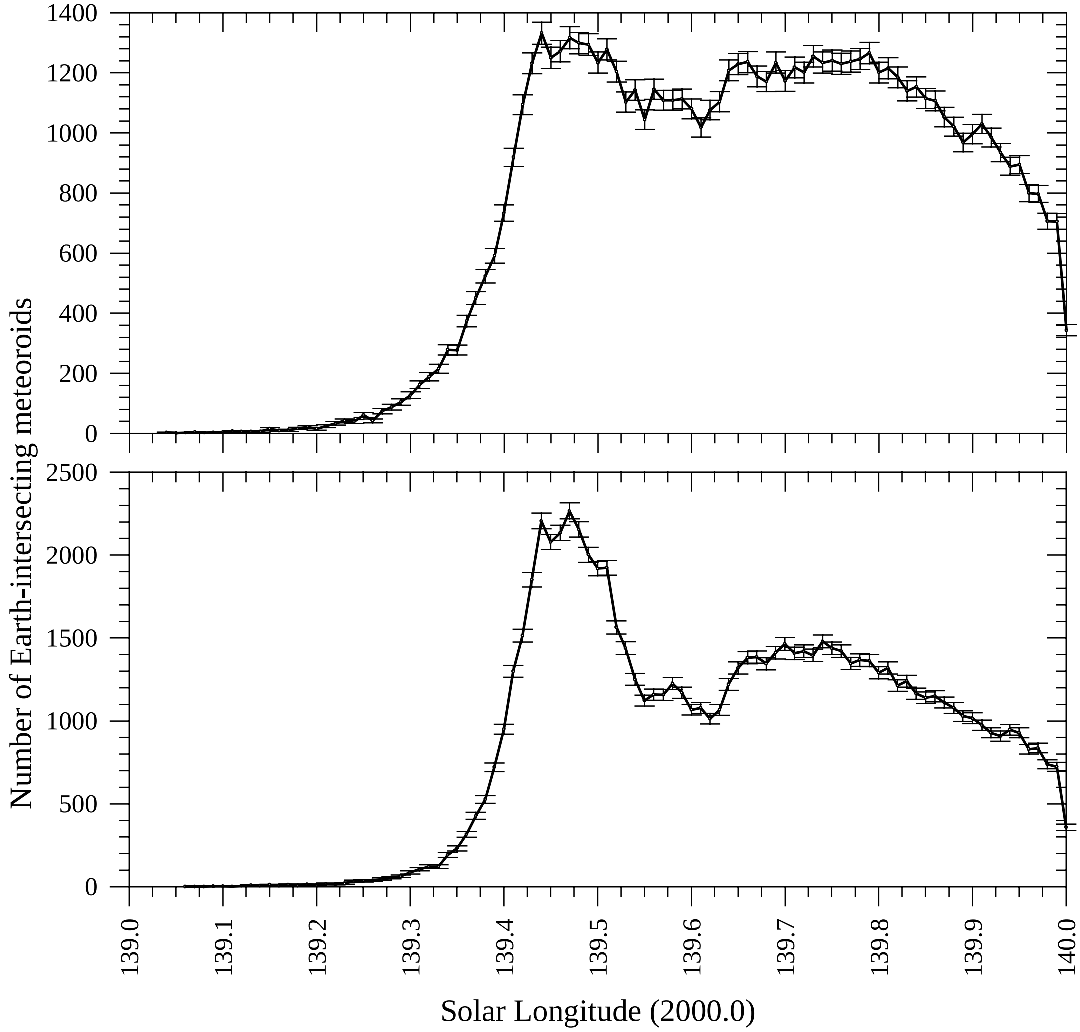

Fig 4.8: Cumulative activity as a function of solar longitude for model 22 (top) and model 33 (bottom) from the past 2000 years of ejections from Swift-Tuttle at the present epoch.

The model peaks generally follow closely to one another. A glaring exception here is model 11, which shows marked deviation from the other models and the observed peak locations. This anomaly may in part be explained by the relatively small number of meteoroids from this model in several of the examined years. As well, the ejection conditions for this model (low density meteoroids, with low ejection velocities) may be unrealistic. The distribution of variances of fit between the predicted and the observed times of maximum are quite small for all models (except 11), with the best overall fit being due to model 21. Indeed, model 21 is the only model which agrees with the observed times of peak within error for all eight years, except 1993.

The coefficient of relative fit for the activity profile at Earth each year and for each model is also given in Table 4.2 in parentheses after the total number of test meteoroids encountered in a given year. This value is found from subtraction of the observed outburst profile for each year from the normalized number of test meteoroids found in every equivalent solar longitude bin (to a resolution of 0.01°) from 139°-140° and summation of the squares of the difference between the observed and theoretical profile in this interval. Note that the difference in fit between years is not generally significant owing to differing numbers of observational intervals from year to year with only intermodel comparisons having meaning for one particular year.

The totals in the last row suggest that the ZHR profiles in these years can best be represented by model 22 (Jones ejection velocity with $r^{-0.5}$ heliocentric velocity dependence), though the difference between many models is not large. The exceptions here are model 43 and 11 which have unusually large variances in fit between the observed and theoretical profiles.

4.4.3 Long-Term Evolution (100 000 years)

To study the behaviour of the Perseids over a significant fraction of the lifetime of the stream (variously estimated to be as much as 250 000 years of age (cf. Hughes, 1995), one must first know the orbit of the comet. Unfortunately, one cannot, as 109P/Swift-Tuttle has been observed only since 69 BC (Yau *et al.*, 1994). The chaotic effects of random errors in initial conditions imply that the position and ultimately the orbital elements of the comet quickly diverge during backward integrations.

Chambers (1995) investigated the long-term motion of Swift-Tuttle both forwards and backwards. He found that the comet's past behaviour implied a Lyapunov exponent of approximately 180 years in the immediate past and its current and future motion to be influenced by the 1:11 libration Swift-Tuttle currently experiences with Jupiter.

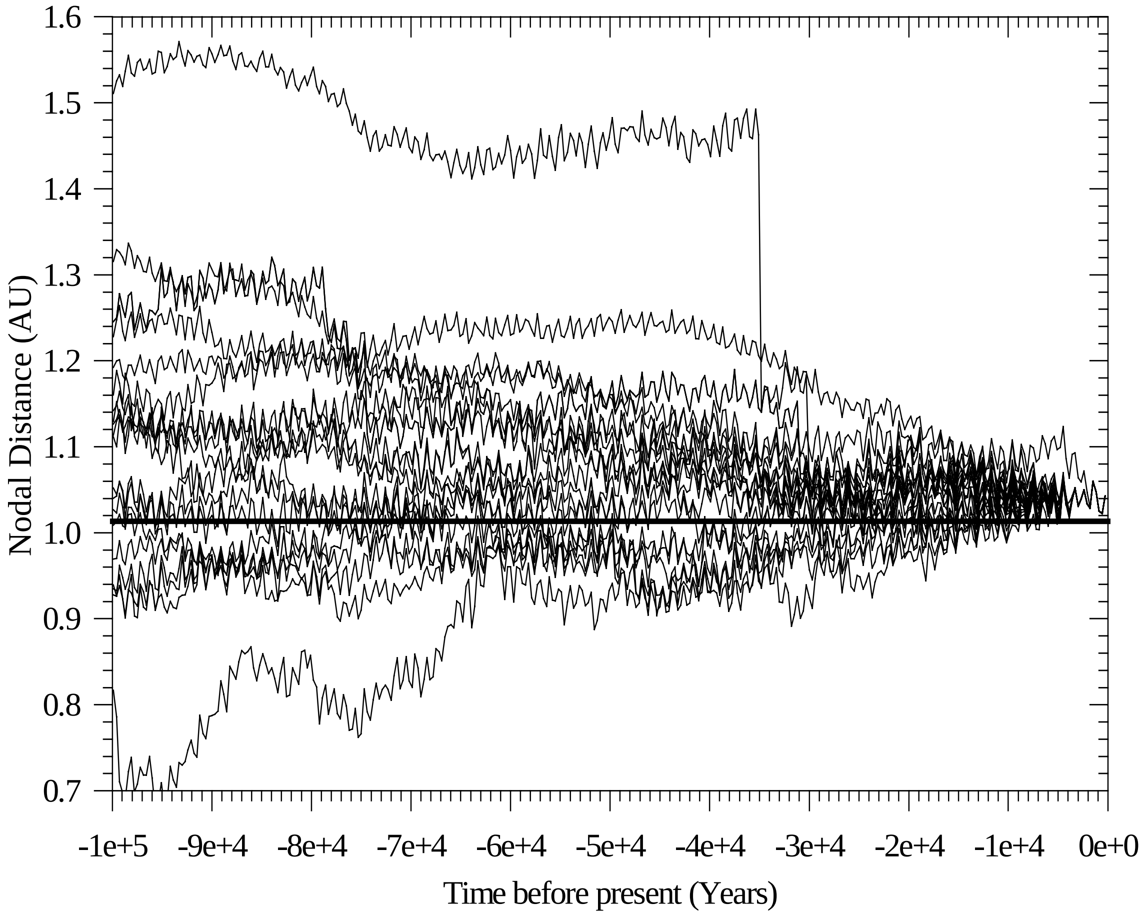

Fig 4.9: Nodal distances of 20 cloned variations of 109P/Swift-Tuttle integrated backwards starting from the present 100 000 years (see text for more details).

To attempt to model the stream, we generated plausible past orbital elements for the comet by taking the six-vector of the comet at perihelion in 1862 and “cloning” 20 different seed orbits about the nominal position of the comet within a sphere of radius 10 km (comparable to the size of the nucleus of the comet). Each seed orbit was then integrated backward in time using the SWIFT symplectic integrator (cf. Levison and Duncan (1994)) with a timestep of 0.25 days for 100 000 years using the JPL DE404 ephemeris to generate all initial planetary positions and velocities. Of greatest importance to the visibility of the Perseid stream on Earth present is the distance of the descending node of the comet from the Earth’s orbit (based on our earlier results from Fig 4.4). This is shown at 300 year increments for all 20 cloned orbits for the full integration time in

Fig. 4.9. The general position of the node over this time is remarkably close to the Earth, a result also found by Chambers (1995). Indeed, for the last 20 000 years no nodes are found outside $0.9<R_d<1.15$, a similar finding to Chambers (1995). From the ensemble of 20 cloned orbits, two orbits were chosen at intervals of approximately 5000, 10 000, 20 000, 50 000, 75 000 and 100 000 years. The two orbits were selected to be the most "extreme" from the set in the sense of having the largest or smallest semi-major axis. The orbital elements used for each of these two seed orbits (1 for the lower values and 2 for the larger values of *a*) are given in Table 4.3. Using these input orbits, a full set of test Perseid starting orbits was generated using a model 42 variant (which was felt at the outset to be most representative) for ejection velocities as with the shorter-term integrations. By comparing the final results of these simulation runs, we hope that some indication of the importance of the cometary starting orbit and thus the probable error in the simulation can be inferred, given that the true orbit from this long ago is not known a posteriori.

The final distributions of meteoroids at the present time show much less temporal variation than did the test particles from integrations over the last 2000 years as might be expected. Even ejections only 5000 years of age show a surprisingly constant annual level of activity with an average of roughly 30 earth-intersecting meteoroids encountered per year. Some small periodic variations in the annual influx from orbit #1 for ejection 5000 years ago is evident and is possibly attributable to the accumulated effects of Jovian impulses (see Sect. 4.5.). The number of Earth-intersecting test meteoroids drops off nearly linearly in time for orbit #2, but much more slowly for orbit #1 particles. This effect might be attributable to the node of orbit #1 being inside the Earth's orbit for more recent cometary starting orbits resulting in easier delivery of meteoroids to Earth as radiation pressure preferentially moves the nodes (on average) further outward.

The distribution in solar longitude of meteoroids for older ejections is given in Table 4.4. The locations of the maximum for long-term ejecta at the present epoch, found by fitting a gaussian to the present distribution of modelled meteoroid nodal longitudes, shows a slight decrease in position with age, the maximum position following $\lambda_o=(141.05 \pm 0.08) - (3.23 \pm 1.23) \times 10^{-5}$ Y.

Ejection	a	e	i	ω	Ω	T
5000	25.21880	0.9605500	114.755	151.850	137.475	-6990.0
	28.14470	0.9656100	113.121	152.250	138.321	-6990.0
10000	24.59700	0.9617600	114.866	152.909	136.251	-11989.0
	30.34540	0.9685600	112.882	153.146	136.833	-11989.0
20000	22.97440	0.9567600	116.881	150.168	132.322	-21990.0
	31.78580	0.9691800	113.291	156.677	133.632	-21990.0
50000	22.06170	0.9574700	118.929	156.287	114.114	-51990.0
	37.59280	0.9714600	111.332	154.537	128.582	-51990.0
75000	21.40130	0.9495200	123.386	150.150	108.858	-76990.0
	39.68530	0.9728500	111.641	156.933	124.972	-76990.0
100000	20.72780	0.9357000	120.977	175.776	83.694	-101980.0
	49.09810	0.9808400	113.213	163.645	119.646	-101980.0

Table 4.3: Initial seed orbits (1 and 2) for Perseid integrations from 100 000 years to the present at the intervals (before present) shown in the first column. All angular elements are J2000.0; the final column is the epoch of perihelion in units of years before the present.

This relation would imply that the rate of nodal progression is very similar for all ejecta and the parent comet up to 5000 years ago. This relation also explains the asymmetry in the broad rate profile of the shower, namely that past ejections accumulate in the region 139°-141° with the older ejections occurring predominantly in the earlier portions of this interval. Note that this relation does not take into account the position of current ejecta maximum (more recent than ~6000 years ago) which is located closer to the comet's current nodal longitude than the much older ejecta and peaks roughly 1.5 degrees earlier than the above relation would suggest.

The gaussian half-width of the nodal distribution profiles of earth-intersecting meteoroids at the present epoch follows the relation

$$W = (0.774 \pm 0.550) + (9.183 \pm 0.830) \times 10^{-5}\, Y \tag{4.9}$$

Time since Ejection	λ_{peak}	Width
6990.0	140.56°±0.02°	1.70°±0.02°
	140.85°±0.03°	1.90°±0.03°
11989.0	140.04°±0.04°	1.67°±0.04°
	139.81°±0.08°	2.46°±0.08°
21990.0	141.55°±0.09°	2.30°±0.1°
	139.62°±0.14°	2.80°±0.16°
51990.0	133.31°±0.11°	4.59°±0.11°
	140.13°±0.32°	7.36°±0.32°
76990.0	146.99°±0.26°	8.00°±0.26°
	139.39°±0.42°	8.95°±0.42°
101980.0	122.99°±0.86°	15.31°±0.95°
	136.00°±0.78°	11.07°±0.78°

Table 4.4: Solar longitude locations and widths of maxima for each ejection for Earth intersecting Perseid meteoroids at the present epoch for seed orbits 1 and 2.

This demonstrates how the stream can be so long-lived at the current epoch given even a modestly long age, with ejections 100 000 years ago currently having full widths of nearly 25 degrees in solar longitude.

The development of the stream over the last 100 000 years is summarized in Fig. 4.10 where nodal positions of test meteoroids at the present epoch are presented. The central portion of the meteoroid nodal footprint of the stream always remains very close to the Earth for both orbits and all masses. The nodal distribution formed from orbit #2 shows considerably more elongation than orbit #1, reflecting the higher eccentricity and semi-major axis of the latter orbit and the large number of test meteoroids which move into sungrazing and near-sungrazing orbits.

4.5 Discussion

The above results suggest the models used are not unreasonable representations of the actual ejection process of 109P/Swift-Tuttle one that is undoubtedly more complicated than our very simplified ejection schemes. In general, the three most reliably measured stream parameters, namely the activity as a function of solar longitude per year and variations in peak activity from year to year as well as geocentric radiant distributions of shower meteors, are consistent with the modelling results within the limitations of both.

The investigation of the change in the final distribution of Perseid activity seen at Earth with variations in cone angle has revealed simply that the narrower cone angles tend to concentrate the resulting meteoroids more closely to the original comet nodal locations for recent ejections. Over periods of order five revolutions, the effects of narrower cone angles become masked as planetary perturbations begin to dominate the dispersion of the stream.

The one major remaining discrepancy between the modelled results and the actual observations which remains is the one to two hour difference in peak time for the 1993 and 1994 Perseid outburst maxima. There are two possible explanations for the differences. One would be that material associated with the outburst in 1993 and 1994 is richer in older ejections, implying that the comet was particularly active in 1610 or 1479, the two passages other than 1862 which our simulations suggest should contribute significantly to the outburst portion of the stream in these years. The ejecta from both of these passages would place the nodal longitude of the peak roughly 0.1° later than what is currently given by the models and could explain the discrepancy.

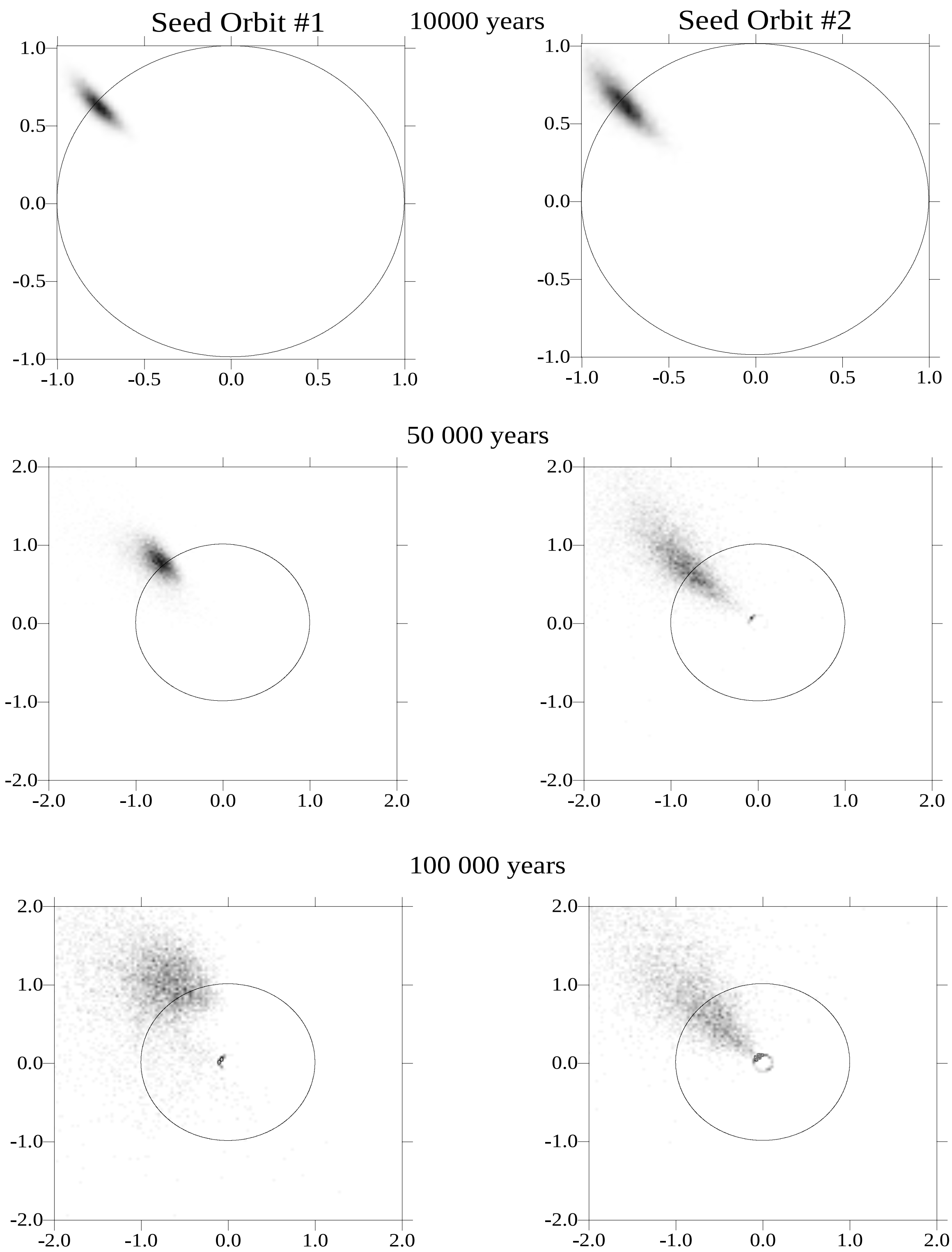

Fig 4.10: Descending nodal distribution of all Perseid meteoroids of mass 0.1g (for model 42 this means $\beta=2\times10^{-4}$) with the ages shown for both initial seed orbits. The circular outline is the orbit of the Earth and all measurements are in A.U.

The geometry of the comet's passage in 1610 and 1479 placed it well below the likely detection threshold for visual observations (Yau et al, 1994) and the fact that no observations exist for either of these returns suggests that the comet was not intrinsically brighter than its long-term average. Alternatively, the ejection geometry in 1862 might have been much more collimated than the rather broad, hemispherical ejection geometry adopted. In particular, for ejections with a substantial velocity component normal to the cometary plane, it is possible to change the mean nodal longitude as much as 0.1-0.2° with normal "Whipple"-sized ejection velocities. More precisely, the change in nodal longitude can be described by (Roy, 1978):

$$\Delta\Omega = \frac{r\sin(\theta+\omega)}{na^2\sqrt{1-e^2}\sin i}\Delta V_n \tag{4.10}$$

where n is the mean angular velocity ($2\pi/T$), θ the true anomaly and ΔV_n is the component of the velocity normal to the orbital plane such that the object is seen to orbit in the counterclockwise direction as seen from this pole. Thus to increase the nodal longitude from the initial ejection velocity alone requires a positive value for ΔV_n. Fortunately, detailed observations from the 1862 passage of Swift-Tuttle exist and these have been examined in detail by Sekanina (1981). In particular, he reconstructed the velocity vectors of the major jets near perihelion based on observations of fans and other structures visible to Earth-based observers during that passage. Over the two month period nearest perihelion, it was found that some 70% of all observed ejections had a velocity component with positive ΔV_n.

Fig 4.11 shows the change in the osculating node for the Perseids as a function of the normal component of the ejection velocity (V_n) and the ejection position along the orbit. For ejection pre-perihelion at a modest distance from the sun (r>1.5 A.U.), a velocity of less than 50 m/s is needed in the normal direction to produce a positive shift of 0.1° in the nodal longitude. This is well within the allowable range of ejection velocities for visual-sized meteoroids using the normal Jones/Whipple ejection model for a comet the size of Swift-Tuttle and suggests that the activity from 1993 and 1994 might best be explained by pre-perihelion ejection from isolated sites residing at latitudes significantly different from the sub-solar point. Indeed, Sekanina (1981) noted that "..the net momenta exerted on the nucleus by ejecta from the active areas in 1862 were virtually all directed to the south of the orbital plane.", implying that almost all ejections had a strong northward (positive V_n) component.

Perseid photographic data, representing roughly 600 orbits according to Lindblad and Porubcan (1994), also contains detailed distributions of all

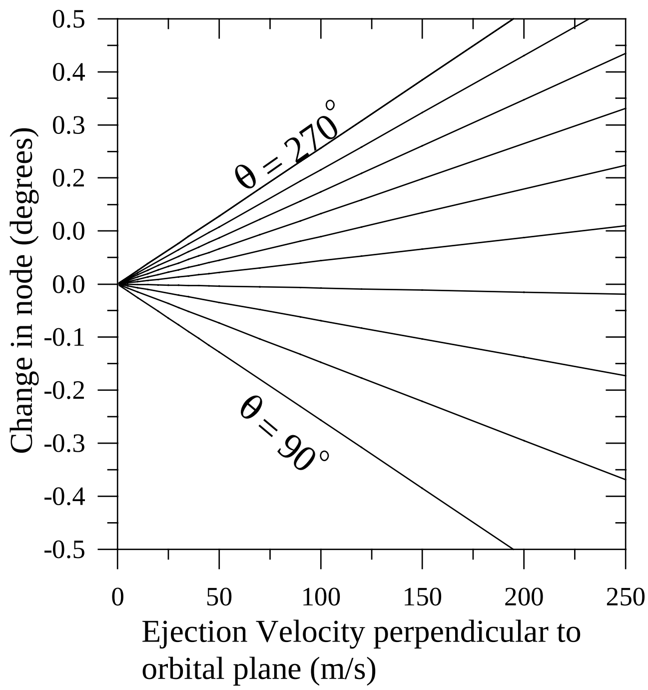

Fig 4.11: The change in the osculating nodal longitude at ejection for meteoroid test particles as a function of the normal component (V_n) (relative to the cometary orbital plane) of the initial ejection velocity and true anomaly (θ) at ejection. Each line represents values for the true anomaly from 270°-90° in steps of 20°.

orbital elements. However, the previous discussion concerning large errors in semi-major axis, for example, applies to lesser degrees to the errors for many other orbital elements and renders their usefulness questionable. The original data sources from whence these orbits are extracted often do not list estimates of the errors in other elements for individual orbits. An examination of the dispersion in mean elements from the simulation output yields standard deviations less than 0.003 A.U. in q, 0.5° in inclination, and 0.6° in the argument of perihelion for the combined ejections over the last 2000 years. For comparison, Spurny (1995) lists detailed data (and errors) for 27 Perseids photographed with fish-eye cameras during the 1993 Perseids. His distributions show average errors of 0.005 A.U. in q, 1.1° in inclination and 2.4° in the argument of perihelion. In all cases the average errors are 2-4 times the maximum dispersion in the cumulative theoretical distributions for the same elements. Porubcan (1977) examined most of the presently available Perseid orbits and showed that there are significant intersurvey differences in dispersion among various photographic datasets. He concluded that the observed dispersions are greater than the true dispersion in the stream, a conclusion we also have reached. Of the several hundred Perseid orbits available, there is a small number of very precise orbits with errors smaller than our expected dispersions; in this case, however, the number of usable orbits drops to a one to two dozen and thus no statistically meaningful comparisons can be made. We do not treat photographic orbital elements further and discuss only geocentric radiant distributions in the remainder of this work.

The considerable evolution experienced by some Perseid particles, particularly the changes in the argument of perihelion over time periods of order 50 000 years, resulted in movement of the ascending node of some test meteoroids to Earth-intersection. The result was a shower of duration two to three weeks which occurs in mid-March from the southern hemisphere. Table 4.5 provides orbital details of this theoretical twin shower of the Perseids, along with drift of the radiant point and spread in the radiant. A search for showers possibly associated with this theoretical radiant yielded two with close similarities: the Gamma Normids and the Theta Centarids (Jenniskens, 1994). Both have radiant positions very close to our expected location and peak at very nearly the same nodal longitudes expected for the Perseid southern shower. The lack of velocity information for these streams means that the values for a, e and q are uncertain; within uncertainties the showers might be linked to the southern Perseid radiant. The Theta Centarids, in particular, show similarity to the theoretical stream and it would be most interesting to get accurate velocity information for these streams to test for any association.

Stream	a	e	i	ω	Ω	q	α	δ
Theoretical Southern Perseid Twin	21±3	0.99±0.01	121±21	76±24	165±22	0.61± 0.19	220	-43
γ Normids	∞	1.0	133	41	172	0.89	249	-51
θ Centarids	∞	1.0	128	27	153	0.90	210	-41

Table 4.5: Orbital elements and radiant location for the theoretical Perseid southern twin (at ascending node) and the same for two observed showers with comparable elements and radiant locations in mid-March (from Jenniskens (1994)).

4.5.1. Planetary Impulses on the Perseid Stream.

The planets Jupiter and Saturn pass within 1.6 and 0.9 A.U. respectively of the orbit of 109P/Swift-Tuttle. The comet's high inclination is usually invoked to suggest direct planetary perturbations on the stream to be minimal and the stream quite stable. Over long time periods this is certainly true as most stream meteoroids have moved in essentially the same general orbit as Swift-Tuttle for many thousands of years, a result confirmed by our direct integrations and others (cf. Hamid 1951).

However, as the Perseid stream is a continuous ring of meteoroids, some meteoroids always experience the maximum direct perturbations from either Jupiter or Saturn. Since at the present epoch the descending node of the parent comet is only very slightly outside the Earth's orbit (0.004 A.U. outside for the 1862 passage), even small perturbations can move Perseid meteoroids from non-intersecting to Earth-crossing orbits.

In general, a Perseid meteoroid passing some distance from a planet will experience an impulse that changes its orbit by a small amount. This small perturbation results in a significant change in a and e since the orbit of 109P is nearly unbound. As the stream orbit does not pass close to any of the outermost planets (minimum distances from Uranus and Neptune are 2 and 6.5 A.U. respectively), only Saturn and Jupiter are important in this regard. Fig 4.12 shows the envelope of closest possible distances between Jupiter and Saturn and the mean orbit of Swift-Tuttle. Any actual encounter between a Perseid meteoroid and one of these planets will have a planet-meteoroid distance curve inside these envelopes and with larger curvature. A typical encounter between Jupiter and a Perseid meteoroid is also shown in Fig. 4.12 (thin line).

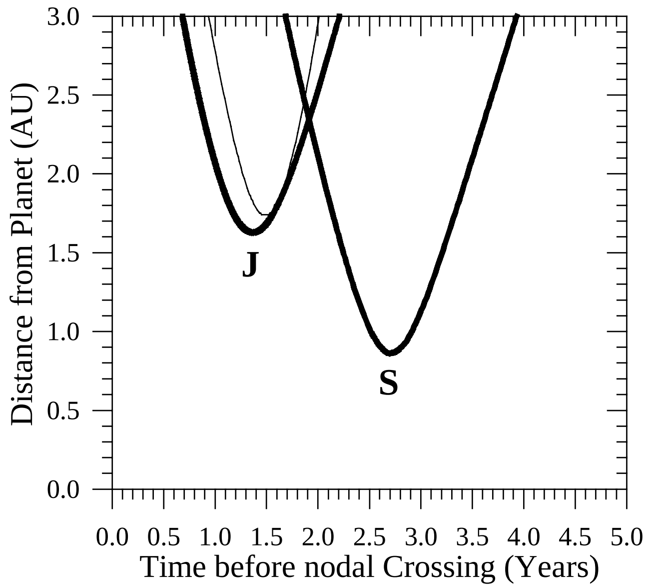

Fig 4.12: Closest approach distances between the mean Perseid orbit (taken as the osculating orbit of 109P at its 1862 perihelion passage) and the planets Jupiter and Saturn (shown as bold lines) as a function of the time before nodal passage. The change in distance between Jupiter and a typical Perseid meteoroid is also shown (thin line).

For Earth-encounter, the radius of the descending node must equal the Earth's orbital distance from the sun. In general the descending nodal radius in A.U. (R_d) is given by

$$R_d = \frac{a(1-e^2)}{1-e\cos\omega} \tag{4.11}$$

where ω is the argument of perihelion. The change in the nodal radius due to variations in the individual osculating elements is given by

$$dR_d = R_d \frac{da}{a} + \frac{e}{1-e\cos\omega}\left[a(1-2e) + R_d \cos\omega\right]\frac{de}{e} + \frac{R_d e \sin\omega \; d\omega}{1-e\cos\omega} \tag{4.12}$$

In an encounter between a planet (in this case Jupiter or Saturn) and a Perseid meteoroid on a retrograde orbit crossing the planet's orbit above the ecliptic plane with dominant motion perpendicular to the planet's orbit and inward, the net impulse is always a positive one and increases the energy of the associated meteoroid. The result of this effect is that the impulse delivered by Jupiter and Saturn produces a net inward shift in the node of perturbed Perseids. This shift results from the fact that the perturbation decreases the effective perihelion distance of the orbit. Physically, the effect can be understood once it is seen that the encounter with either of Jupiter or Saturn will rotate the velocity vector toward the ecliptic plane. It is precisely this effect which causes the inward shift of the node of meteoroids visible in Fig. 4.3 by a maximum amount of approximately 0.01 A.U. It is not possible to use an Opik-like (or two-body) formalism to describe this encounter with Jupiter as the closest approach distance is almost 5 Hill Sphere radii from Jupiter and the impulse occurs over an extended region where the meteoroids' heliocentric velocity changes appreciably (cf. Greenberg *et al.*, 1988 for a discussion of two-body encounters).

We have investigated this effect through numerical simulation and find that virtually all of the impulse causing this change occurs during the short interval of approximately ~1 year on either side of the closest approach to the planet. To verify that this encounter causes the observed nodal shift, we used 5000 test Perseid meteoroids ejected in 1862 and stopped the integration in 1986, mid-way between Jovian perturbations (1979 and 1991). We then used these new elements as starting orbits where each particle was followed with the direct perturbation term for Jupiter present and with it absent. All particles were followed to their descending nodes and the results of the perturbed and unperturbed final orbits compared. In all cases we found the perturbed meteoroids arrived at the node after the unperturbed meteoroids and with smaller nodal radii in the intervals nearest the Jovian closest approaches. The energy difference between perturbed and unperturbed meteoroids in this simulation was greatest for particles having the largest Jovian perturbations, with particles passing closest to Jupiter always found to have larger energies than the equivalent unperturbed trajectories. Fig 4.13 shows the relative energy difference between meteoroids experiencing close approaches to Jupiter relative to those which do not. Note that the local maximum near 2008 is an artifact owing to the inclusion of the perturbations from Saturn during its 2006 close approach to the stream.

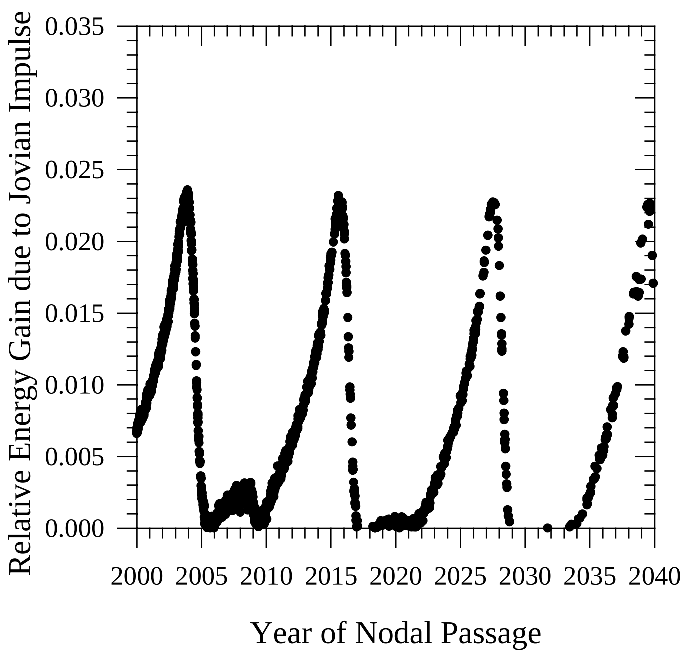

Fig 4.13: Change in the energy of Jovian perturbed meteoroids relative to unperturbed Perseid meteoroids as a function of the time of their nodal passage.

The magnitude of the perturbation in nodal radius is almost exactly the same for Jupiter as for Saturn, the net gravitational impulses for closest approach Perseids being identical owing to the closer distance of approach to Saturn (1.77 times) and slightly longer impulse time (for Saturn perturbations) precisely compensating the factor of 3 lower mass for Saturn.

Since 109P/Swift-Tuttle has had a nodal point outside the Earth's orbit for the last several thousand years, most meteoroids from these recent ejections are not accessible to Earth. On average, we have found that for our simulations the mean effect of radiation pressure is to move the node slightly further outward, though this is not strictly the case for any one Perseid meteoroid, the final difference being a function of the initial ejection distance, velocity and particularly subsequent planetary perturbations for any given test particle. Only impulsive perturbations from Jupiter and Saturn can cause enough change in nodal distance for recently ejected meteoroids to make them visible at Earth.

This effect should produce noticeable changes which may persist for several years in the activity of the stream over restricted intervals in solar longitude every 12 and 30 years. This activity may be further heightened by the "focusing" effects of the perturbation, which concentrates the otherwise scattered nodal points of individual meteoroids, a direct result of the impulsive effects being larger than the smearing effects of initial ejection velocity and ejection geometry for recent ejecta. The close approaches by Jupiter and Saturn to the stream and an observed inward shift in the nodal positions of meteoroids show a lag of 1-3 years and a comparable duration (see Fig. 4.3). Table 4.6 lists the dates of close approach to the stream by Jupiter and Saturn over an interval of one century.

That the position of the planets might affect the observed shower activity on Earth is not a new idea. Guth (1947) suggested that some showers were prone to increases in activity when the stream's orbit was in conjunction with a major planet. More recently, Jenniskens (1997) has shown that many streams show outbursts preferentially when the positions of Jupiter and Saturn are near conjunction with the stream. We suggest that in these cases an impulse effect similar to the one found for the Perseids is also at work.

Table 4.6: Dates of closest approach between Jupiter and Saturn and the Perseid stream over the interval 1860-2050.

Jupiter Closest Approach Date (YY/MM/DD)	Saturn Closest Approach Date (YY/MM/DD)
1860/9/15	1889/1/1
1872/7/26	1918/6/13
1884/6/5	1947/11/21
1896/4/18	1977/5/4
1908/2/27	2006/10/8
1920/1/7	2036/3/24
1931/11/17	
1943/9/27	
1955/8/7	
1967/6/17	
1979/4/26	
1991/3/6	
2003/1/14	
2014/11/24	
2026/10/4	
2038/8/14	

4.5.2 Geocentric Radiant Distributions - Theoretical vs. Observed.

The distribution of the theoretical radiants for the full 2000 year and 100 000 year integrations are shown in Figs. 4.14 and 4.15 for photographic sized meteoroids (10g<m<0.1g). The temporal change in the rms width of the cumulative radiant distribution as a function of time for both orbit #1 and orbit #2 is shown in Fig. 4.16. The radiant dispersion for older ejections was approximated by weighting each geocentric radiant from an older ejection by the time between the next most recent and next oldest ejection in the model divided by the mean period of the comet.

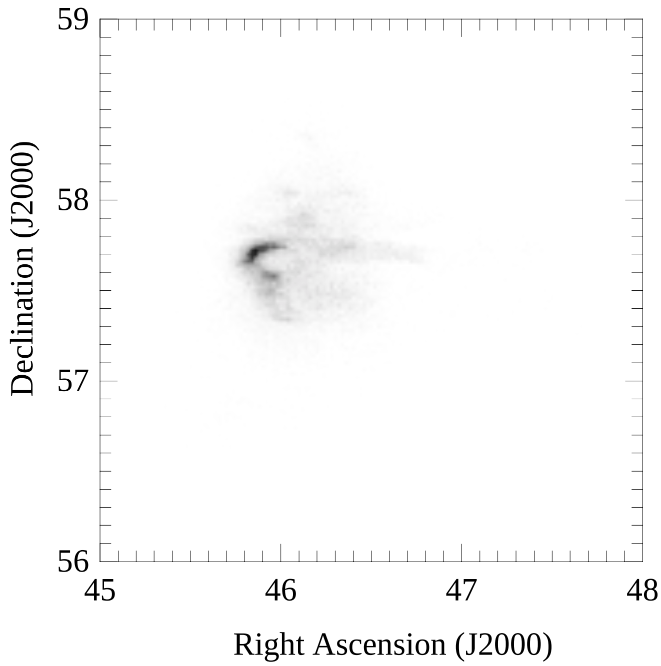

Fig 4.14: Geocentric radiant distribution for all Earth intersecting Perseids ejected 59 to 1862 A.D. at the present epoch for photographic-sized meteoroids (mass > 0.1 g) from model 42. Grid resolution is 0.02°. The dynamic greyscale range for this binning is from 0 to 320.

While some difference exists between the dispersions found from orbit #1 and #2, the most consistent relation for the dispersion of the Perseid radiant over the full 100 000 years using the average of both orbits is

$$W = (4.74 \pm 0.84) \times 10^{-3} Y^{0.55} \tag{4.13}$$

where W is in degrees and Y in years. The exponent in this power-law is very close to the 0.5 expected for the case of random-walk-type diffusion.

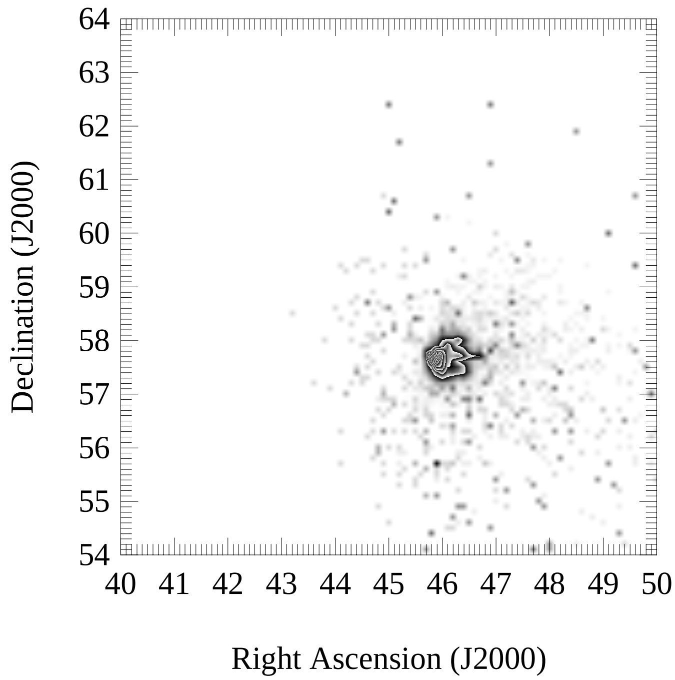

Fig 4.15: Geocentric radiant distribution for all Earth intersecting Perseids ejected over the past 100,000 years at the present epoch for photographic-sized meteoroids (mass>0.1 g) for model 42. The dynamic greyscale range for this binning is from 0 to 350.

The observed radiant dispersion for the Perseids changes as the Earth passes through the stream. Kresak and Porubcan (1970) investigated the radiant of the stream using 250 photographed Perseids. They found the radiant showed a significant change in size across the stream, with the average dispersion being 1.39° for λ_o<139°, 1.10° for 139°<λ_o<140.3° and 1.33° for λ_o>140.3°. A more recent examination of the same question by Lindblad and Porubcan (1995) revealed a similar trend. While this trend is often interpreted as suggestive of older material outside the core portion of the stream (an observation supported by our findings), it is also significant that material further from the core of the stream has been, by definition, more affected than Swift-Tuttle by planetary perturbations and is thus more dispersed. Fig 4.17 shows the dispersion at the present epoch for individual ejections in the intervals before, during and after the main maximum. It is clear there is a large increase in dispersion away from the core of the stream for ejections of the same age.

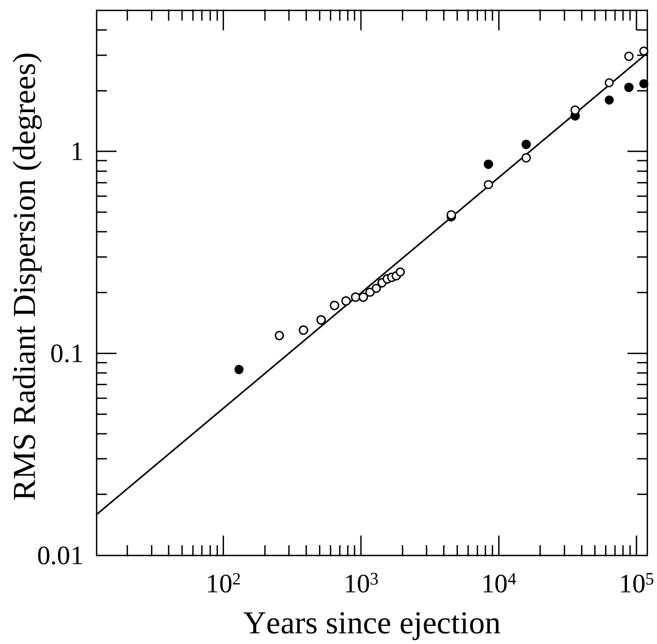

Fig 4.16: Change in the rms width of the Perseid radiant for cumulative ejections over the past 100 000 years for seed orbit #1 (filled circles) and seed orbit #2 (open circles).

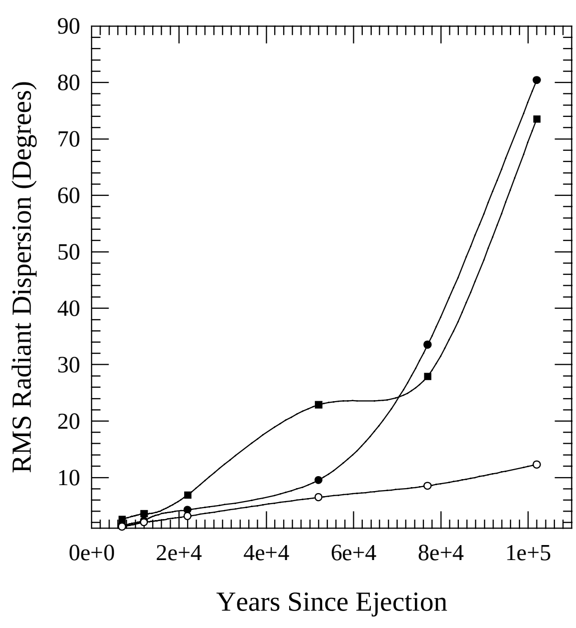

Fig 4.17: Radiant dispersions for individual ejections of photographic-sized meteoroids from 5000-100,000 years ago for Perseids in the pre-maximum period (λ < 139°)(solid circles), the maximum period (139°<λ<140.3°) (open circles), and in the post-maximum region (λ>140.3°) (solid circles).

Whipple and Wright (1954) noted a strong correlation between the nodal width of a stream and radiant dispersion. They also noted that the change in scatter as a function of mass should indicate whether physical forces such as initial ejection velocity and radiation effects are dominant over planetary perturbations. In Sect. 4.2 it was shown from an examination of visual-sized meteoroid radiant spreads from all models over the last 2000 years that the absolute rms size of the radiant is dominated for the first few revolutions by the initial ejection velocity and later affected by radiation pressure, whereas the rate of change of the radiant size is similar for all initial ejection conditions and densities of meteoroids and hence controlled by planetary perturbations (see Fig. 4.6). In Fig. 4.18 the radiant dispersion for faint visual and radar class meteoroids (10^{-3} g<m<10^{-5} g) is shown for comparison to the photographic class meteoroids from the same models for orbit #2. In general, the radiant dispersion at present from any past ejection over this period tends to be greater for the smaller meteoroids than for the larger ones, but the variation of the change between the two mass categories is similar for each period of activity of the stream. This supports the earlier conclusions of 4.2.

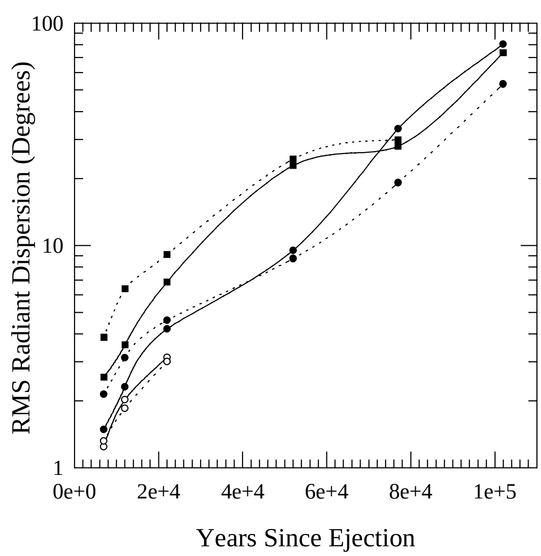

Fig 4.18: Radiant dispersion of faint visual and radar class meteoroids (dotted line) as compared to brighter photographic Perseids for ejections from 5000-100 000 years ago. Symbols have the same meaning as in Fig 4.17; only ejections with at least 20 representative Earth intersecting members at the present epoch are included.

Lindblad and Porubcan (1995) found that the radiant area increased as the magnitude of the photographic Perseid decreased. Porubcan (1973) noted the telescopic radiant spread of the shower to be significantly larger than the photographically determined width. All of these observations are consistent with our results showing the radiant spread to generally be larger at the present time for smaller meteoroids.

The average position of the geocentric radiant for photographic sized meteoroids from ejections over the last 2000 years is at α=46.1° ± 0.1° and δ=57.66° ± 0.05° referenced to J2000.0 and solar longitude 139.7°. This compares well to the location of the “new” component of the stream (outburst portion) found by Lindblad and Porubcan (1995) at α=46.85° ± 1.8° and δ=57.6° ± 0.99°.

4.5.3 Progression Rate of the Node.

The orbits of the Perseids and Swift-Tuttle are retrograde, hence the secular perturbations on the stream due to the planets result in a positive increase in the nodal longitude for the shower and the comet.

Hughes and Emerson (1982) have examined the change in position of the peak of the stream from ancient records. They find that since 36 A.D. the node of the stream has advanced at an average rate of $(3.8 \pm 2.7) \times 10^{-4}$ degrees/year on the basis of the reported times of observation of the shower.

To derive a theoretical value for this number, we determined the position of the maximum of ejecta for each mass category at the current epoch for all ejections over the last 2000 years for all models. The slope of this distribution through time is found to be remarkably independent of mass; all masses were found to have an annual nodal progression rate well represented by

$$\frac{d\lambda_{O}}{dt} = (2.2 \pm 0.2) \times 10^{-4} \quad \text{°/year} \tag{4.14}$$

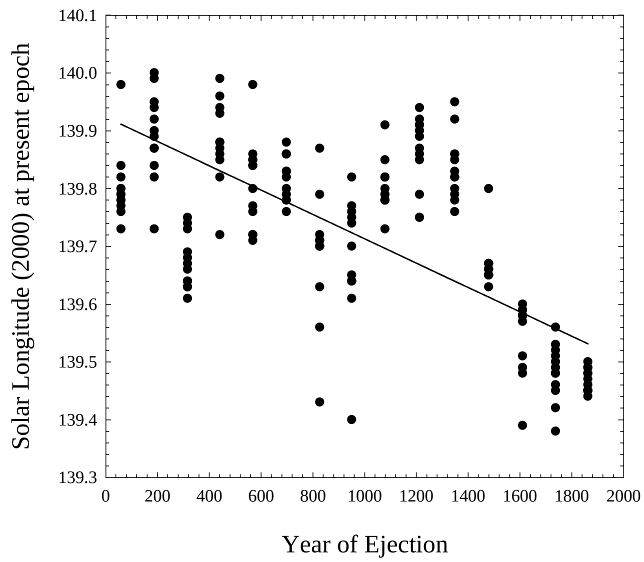

Fig 4.19: Location of the maximum in activity as a function of solar longitude at the present epoch for individual ejections of 0.01g Earth intersecting Perseids over the past 2000 years. The results from all models have been included and each determination of the location of the maximum for each ejection epoch is represented by a single solid circle. The line of best fit is also shown.

Fig 4.19 shows the distribution of maxima as a function of time for 0.01 g Perseids over the last 2000 years.

This nodal progression rate is an order of magnitude larger than the rate found over the interval from 5000<t<100000 years ago (Sect 4.4.3). It is possible the actual progression rate was lower in the distant past as the progression rate would be expected to decrease as we move backward in time if Swift-Tuttle's inclination more closely approached 90°. We note, however, the value of the progression rate at present to be most affected by recent ejections shown to be far more concentrated than older ejections and also more efficient at transporting Perseids into Earth intersecting orbits as the comet's orbit probably passes closer to the Earth than it did in the past. The theoretical progression rate we find is consistent with Hughes and Emerson's (1982) value.

4.5.4 Age of the Stream.

The age of the Perseid stream has remained difficult to determine from past studies. From the nearly perpendicular orientation of the orbital plane, no major perturbations on the parent comet or stream are encountered. From the recent passage of the comet, we know Swift-Tuttle is among the most massive of the Halley-family of comets. Further observations supporting the stream's great antiquity include its very long period of activity and large mass (Hughes and McBride, 1989), estimated to be upwards of 10^{17} g.

That the shower is much older than typical meteoroid streams can be readily inferred simply from its long duration. Southworth (1963), for example, estimated the stream age to be less than 6000 years on the basis of the rate of change in observed elements of photographic Perseids. In the other direction, Katasev and Kulikova (1975) noted that the stream must be younger than the time it takes for Poynting-Robertson drag to cause the particles to collide with the sun, a time of order 10^6 - 10^7 years for visual - sized Perseids. Very few additional attempts to determine the age of the stream have been made.

From the modelling output there are several methods we can employ to estimate the age of the stream.

First, we may use the "average" radiant dispersion and Eq. 4.13. Kresak and Pourbcan (1970) found the mean width of the radiant throughout its period of activity to be 1.27°. This yields an age estimate of $(30 \pm 10) \times 10^3$ years. From a data set with nearly double the number of Perseids, Lindblad and Pourbcan (1970) derived a mean angular dispersion of 1.84° for the entire activity period of the shower which corresponds to an age estimate of $(55 \pm 20) \times 10^3$ years. We note that in both cases these ages represent upper limits as the effects of individual radiant errors are not taken into account in these analyses and thus the true radiant rms spread is smaller than these values.

For the central portion of the stream we attempted to make a direct age estimate on the basis of the current position of the main visual maximum (139.96 ± 0.04°). This was done by summing the activity from each ejection; with each additional passage, the location of the secondary peak in activity (corresponding to the broad maximum as opposed to the outburst maximum) was found. Here we defined such a sub-maximum to be present if the peak in number of test meteoroids in any interval of 0.01° of solar longitude was above the number in all bins between 0.05° before and 0.05° after the position of the local maximum. By doing this for all 15 ejections from 59A.D.-1862 we noticed a slight shift in the position of this maximum as more ejections were added to the total. By assuming the geometry of encounter with Swift-Tuttle has remained reasonably similar to the average over the last 2000 years for the past ~10 000 years (a fact supported by our long term integration of the comet's orbit in Sect. 4.4.3), we can then use this rate of shift, averaged for all models, to extrapolate the number of total ejections needed to produce a peak at 139.96° at present. This procedure was done for all models and the position of the secondary maximum (found to move from approximately 139.7°-139.75° over the whole 2000 year period) as a function of number of ejections added to the total (or equivalently the time) was determined. We note that this produces a lower limit as older ejections add fewer meteoroids to the core portion of the present population (all other things being equal) and each new ejection causes less of a change in the peak position due to the large number of previously existing meteoroids. In this way we find that the shift in maxima would be such as to equal the present location of the observed maximum after $(11 \pm 3) \times 10^3$ years.

We can also use the width of the ZHR-profile at present and compare it to the width of the distributions found for each of the long-term ejections to derive a lower limit for the age of the central portion of the stream, since the width of the individual distributions at present will always be larger than the actual width from cumulative ejections. From Chapter 3, the observed FWHM of the Perseid profile is approximately 2.1 ± 0.1°. Using Eq. 4.9, the ejections attain this width after $(14 \pm 7) \times 10^3$ years, implying that the age of the central portion of the stream must be >7000 years.

The absolute location in (α,δ) of the new and old components of the stream can also be compared with the rate of change in these elements and with the weighted cumulative distribution location for the same elements in order to derive two approximate estimates for the age. Lindblad and Porubcan (1995) have shown that the average radiant location (referenced to λ_o = 139.7° (J2000.0)) is located at α=47.52° and δ=57.96° (from their Eqs. (1) and (2)). From the cumulative distributions over the last 2000 years averaged over all models and referenced to the same solar longitude, the change in right ascension is well represented by

$$\alpha = (45.88 \pm 0.01) + (1.13 \pm 0.03) \times 10^{-4} Y \qquad (4.15)$$

This yields an estimate of $(15 \pm 1) \times 10^3$ Y years for the age of the central portion of the stream.

For the location of the "average" declination for the stream, there is considerably more scatter in the slope of best-fit to the theoretical distribution because the secular variation in the declination is small in comparison to amplitude variations caused by planetary perturbations.

An approximate expression averaged over all models is:

$$\delta = (57.66 \pm 0.01) + (9.3 \pm 3.8) \times 10^{-6} Y \qquad (4.16)$$

which yields a median estimate of ~$(38 \pm 16) \times 10^3$ years. Taken together these two determinations suggest an age of 15-20 000 years as most appropriate.

The above estimates represent the effective age of the majority of the photographic/visual-sized meteoroids in the Perseid stream. The age of the most ancient meteoroids in the stream is much older, the amount of material from older returns having been diffused and hence not contributing significantly to the bulk of the currently visible core population. Perhaps the most effective means of gauging the total age of the stream is by comparing the full nodal spread of the current stream to the theoretical spread. The duration of the visibly detectable stream extends from roughly λ_o = 115° - 150° (from the results of Chapter 3), corresponding to calendar dates from mid-July to late August each year. There are hints that some activity from the shower might be visible outside this boundary, but the levels are lower than can be distinguished using visual observation techniques and we adopt the above as the minimum length of time the shower is presently active.

From Sect. 4.4.2 and 4.4.3, the nodal dispersion from ejections at all visual-sized masses over the last 2000 years remains effectively contained within the region 139-140.5°. Taking Eq. 4.9, the Gaussian half-width from past ejections reaches this full width after nearly 180 000 years, though we caution that this is extrapolated well beyond the region where Eq. 4.9 was determined. If we take a “weak” level of observed activity to be possible even when the mean level of the theoretical activity is at a distance of 2σ from the peak, this would imply an overall age for the stream of ~90 000 years.

4.5.5 Long-Term effects of Terrestrial Perturbations.

Since the earliest recognition of the Perseids in the 19th century, the question of the role of the Earth in the development of the stream has been posed by a number of authors (cf. Twining (1862), Shajn (1923)). Previous works have examined the expected effects based on approximate analytic treatments of the average effect the Earth has on the stream, while ignoring the true physical character of the stream as a collection of many individual particles.

In an effort to address this question directly, we re-ran all long-term integrations using seed orbit #1 with every condition identical, except that the direct planetary perturbations from the Earth were removed. We expect, a priori, that the influence of the Earth will be detected through an increase in the scatter of the orbital elements, particularly, a, i, and Ω in the simulation set containing the Earth as compared to the set without the Earth. The results show that in overall terms the Earth does have a perceptible effect on the evolution of the stream but it is not more than a secondary influence in absolute terms.

That the Earth affects the stream is most evident in the width of the final nodal distributions as shown in Fig. 4.20. Here the difference between the gaussian fit-widths and the final ejections with Earth and without are presented. The influence of the Earth is to add ~10% to the total width of the stream for those points containing the largest number of test particles. Similarly, the radiant dispersion increases by ~10% for any given age of ejection with inclusion of the Earth.

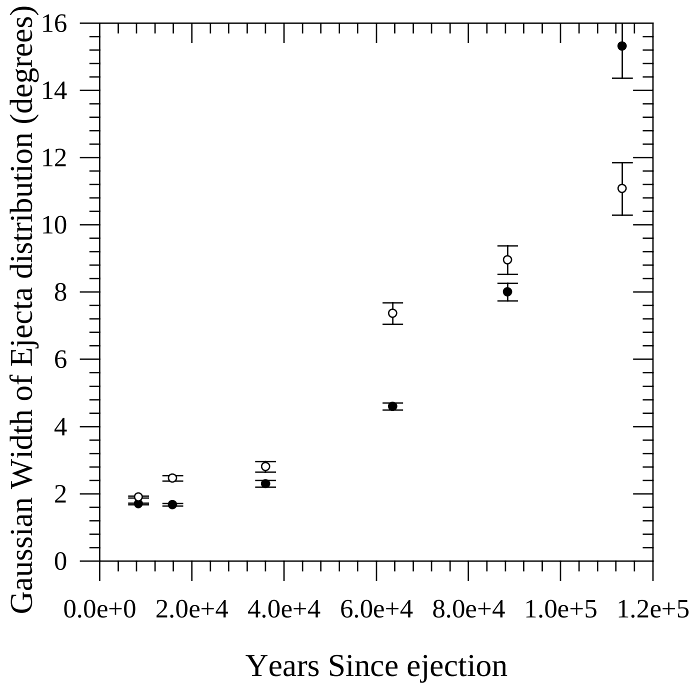

Fig 4.20: The width of the final nodal distribution for Earth intersecting Perseids for orbit #1 with the Earth perturbations (open circles) and without Terrestrial perturbations (solid circles).

The terrestrial effect on the orbital element dispersions is shown in Table 4.7. Here the difference in the rms dispersion in the distribution of a, i and ω for the Earth/No Earth simulations is given as well as the total number of meteoroids used in each distribution. There is a distinct tendency for the dispersions to be lower for the simulation where the Earth is removed (negative values), however the effect is far from universal. Particularly for the oldest ejections where fewer particles are involved, the small statistics overwhelm the relatively minor effect of the Earth's perturbations.

Ejection Time (Year)	Mass grams	# of Meteoroids Earth (No Earth)	Semi-Major Axis (a) (A.U.)	Inclination (i) (degrees)	Argument of Perihelion
5000	10	496 (466)	-0.01	-0.49	-0.11
	0.1	491 (471)	-0.14	-0.54	0.00
	0.001	444 (530	+0.02	-0.53	-0.05
10000	10	400 (383)	+0.1	-0.09	-0.27
	0.1	370 (430)	-0.04	-0.16	-0.26
	0.001	367 (390)	-0.25	-0.8	-0.06
20000	10	253 (303)	-0.09	-0.5	+0.07
	0.1	255 (270)	-0.47	-0.89	+0.29
	0.001	243 (246)	+0.05	+0.42	+0.02
50000	10	198 (212)	-0.7	-0.86	-0.17
	0.1	183 (200)	-1.3	-2.85	-0.83
	0.001	189 (188)	+1.06	-0.18	+0.81
75000	10	87 (88)	+2.15	-2.88	-6.69
	0.1	91 (89)	-1.17	-2.45	+2.32
	0.001	84 (96)	-2.79	-2.68	+0.75
100000	10	40 (49)	-0.17	-3.77	+2.93
	0.1	70 (60)	+0.21	+0.91	+3.98
	0.001	72 (85)	-0.36	+0.41	+2.34

Table 4.7: The difference in Keplerian element rms dispersion of the Perseid stream for seed orbit #1 meteoroids at their descending nodal passage at the current epoch with and without the direct planetary perturbations of the Earth present. The number of meteoroids in each sample is given for the simulations with Earth perturbations present (and without in brackets). The differences represent $\sigma_{No\ Earth}$ -σ_{Earth}. Negative values imply that the presence of the Earth makes the dispersion larger.

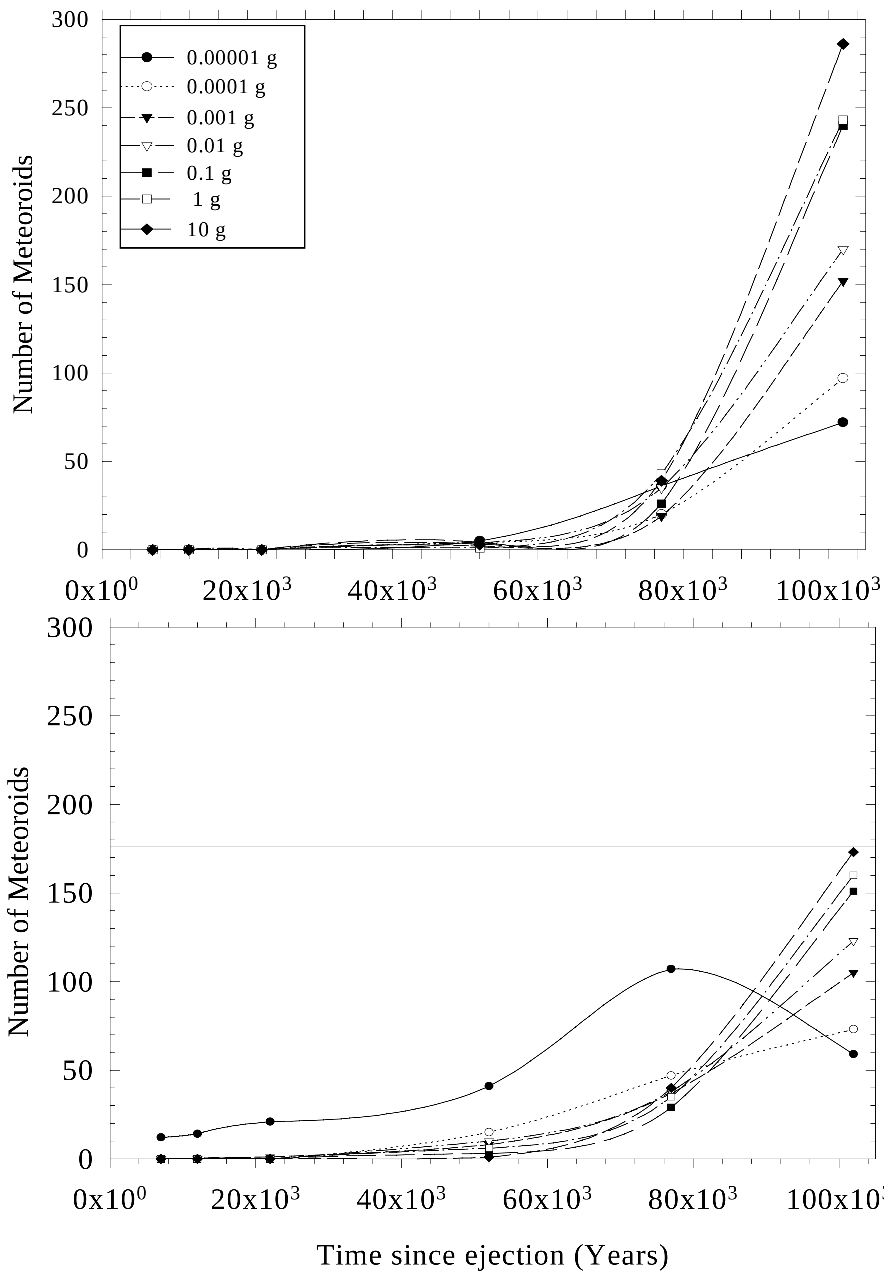

Fig 4.21: Number of Perseid meteoroids which reach a sungrazing state as a function of time since ejection for simulation with the Earth present (bottom) and with it removed (top graph) for all seven mass categories. The legend shows the symbol-mass correspondence. All meteoroids are from model 42 (see text for more details).

When the number of hyperbolically ejected Perseids is examined as a function of time (see Sect. 4.5.6 for more details) in comparison to the number lost without the Earth there is found to be no statistical difference between the two distributions at all masses. This attests to the dominance of Jupiter in ejecting Perseids from the solar system. Curiously, the same comparison of the number of Perseids lost due to attainment of a sungrazing state (when particle gets closer than 0.1 A.U. to the sun) does show a noticeable difference. With the Earth removed it is found that the number of sungrazing states reached is lower for the first 50 000-60 000 years after ejection. The difference is most striking for the smallest mass, where there is a much larger number of sungrazers for all times after ejection right up to 10^5 years. This effect is shown in Fig. 4.21 where the number of sungrazing Perseids is plotted against the year since ejection for simulations with and without the Earth. The Earth plays a more direct role in bringing Perseids to sungrazing states, possibly through the effects of close approaches.

4.5.6 Sinks for Stream Meteoroids: Sungrazers and Hyperbolic ejection

It is usually assumed that the major sink for the Perseid stream is hyperbolic ejection due to planetary perturbations. The effect of collisions in removing meteoroids from the stream has been investigated in detail by Steel and Elford (1986) and they find the survival lifetimes to be at least several million years for Perseid meteoroids, making this a negligible loss channel over the 100 000 year period of our study.

For the long-term integrations, particles were removed from further integration when either their semi-major axis exceeded 200 A.U. or their perihelia decreased below 0.1 A.U., corresponding to a sungrazing end-state. This latter removal condition is likely too strict as several annual meteoroid streams have perihelia inside this distance; the survivability of Perseids this close to the sun is not known, but the evidence from other streams suggests that our sungrazing (or near-sungrazing) conditions should be viewed as upper limits. For comparison, the cometary lexicon typically defines sungrazing states as orbits with perihelia of 0.01 A.U. or less (Bailey *et al.*, 1992).

The fraction of Perseids removed in either of these ways varied dramatically between the long-term orbits #1 and #2. In particular, orbit #2, with a much larger eccentricity and semi-major axis (and hence lower energy) showed an order of magnitude greater loss than orbit #1 for both loss channels.

The primary loss mechanism, especially for smaller meteoroids, was found to be hyperbolic ejection due mainly to direct perturbations from Jupiter with a minor contribution from Saturn. For both orbit #1 and #2 the hyperbolic loss tended to increase as the Perseid mass decreased (and hence β increased), this effect being the result of radiation pressure which increases the average energy of the meteoroid orbit and leads to more losses. However, for orbit #1 this trend was nearly reversed for ejections 10^5 years ago, attesting to the importance of the cometary orbit at time of ejection. After 10^5 years, the percentage hyperbolic loss for orbit #2 for radar-sized meteoroids (10^{-5} g) approached 35% of all ejected meteoroids. For comparison, only 1% of orbit #1 Perseids were lost in any given mass category due to hyperbolic ejection after 10^5 years. Fig 4.22 shows the number of ejected Perseids released at various ejections over the last 10^5 years for all masses for orbit #1 and #2 removed due to hyperbolic ejection before the present epoch.

Bailey *et al.* (1992) demonstrated that comets with orbits nearly perpendicular to the ecliptic plane and perihelion moderately close to the sun (0 - 2 A.U.) are susceptible to sungrazing states. We have found that for larger Perseids ($>10^{-3}$ g) and for both orbit variations used here, our near-sungrazing end state can be almost as efficient as hyperbolic ejection (and in some cases even more so) as a sink for the stream. Fig 4.23 shows the number of Perseids, which enter sungrazing states as a function of ejection time for orbit 1 and 2. The same mass dependence is found as for hyperbolic ejection, with the smallest Perseids being preferentially removed.

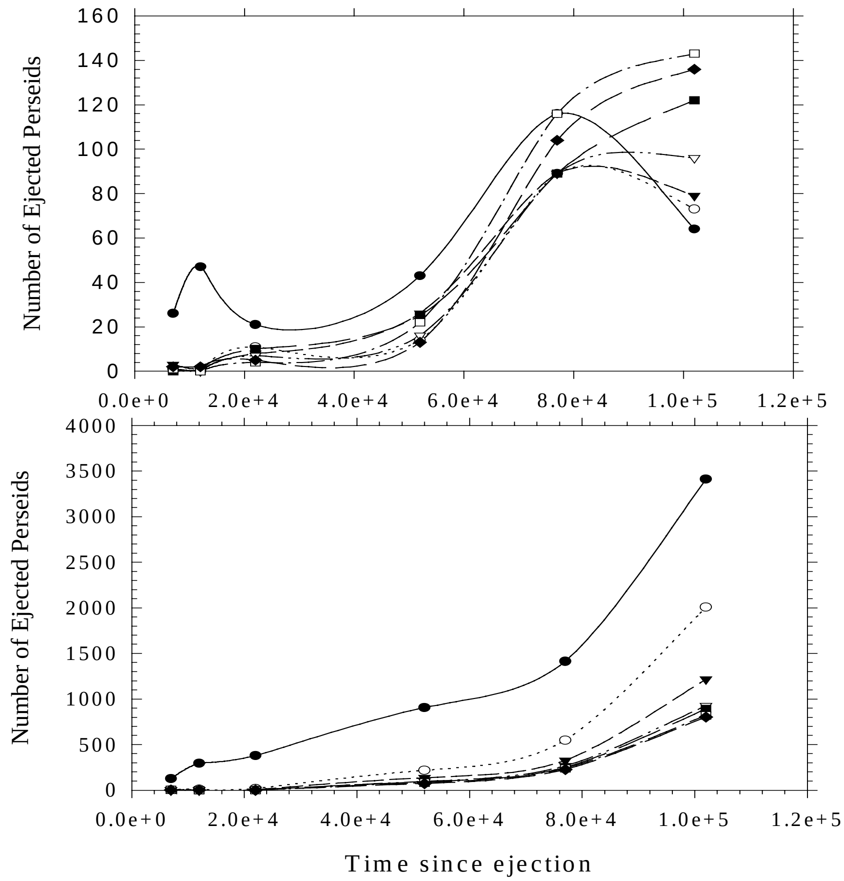

Fig 4.22: The number of hyperbolically ejected Perseid meteoroids as a function of ejection year for seed orbit #1 (top) and orbit #2 (bottom) meteoroids. The symbols are the same as in Fig 4.21.

The length of time needed for meteoroids to enter either of these states depends primarily on the comet orbit adopted (which changes significantly from one ejection epoch to another) for initial ejection from Swift-Tuttle and to a lesser extent on mass. For all but the smallest mass category, the average time taken before any significant number (>10) of Perseids are thrown onto hyperbolic orbits is 40 000 - 60 000 years for both seed orbits. For sungrazing orbits the time taken to reach this state falls in the range from 10 000 - 80 000 years, with an average near 60 000 years. The slope of the number of meteoroids lost as a function of time for either loss channel varies between the two seed orbits, between masses, and times of ejection. In general, a linear or quadratic increase in the number of meteoroids lost is a good representation of the distribution after the initial loss time (as given above), with photographic-sized meteoroids being lost at a peak rate of one to five test particles for every revolution of the comet (corresponding to 0.01-0.05% of the number of total meteoroids initially ejected) after this time from any one mass category due to hyperbolic ejection. This implies a lower limit for the removal time of 50% of the largest particles due to attainment of hyperbolic orbits of ~200 000 years.

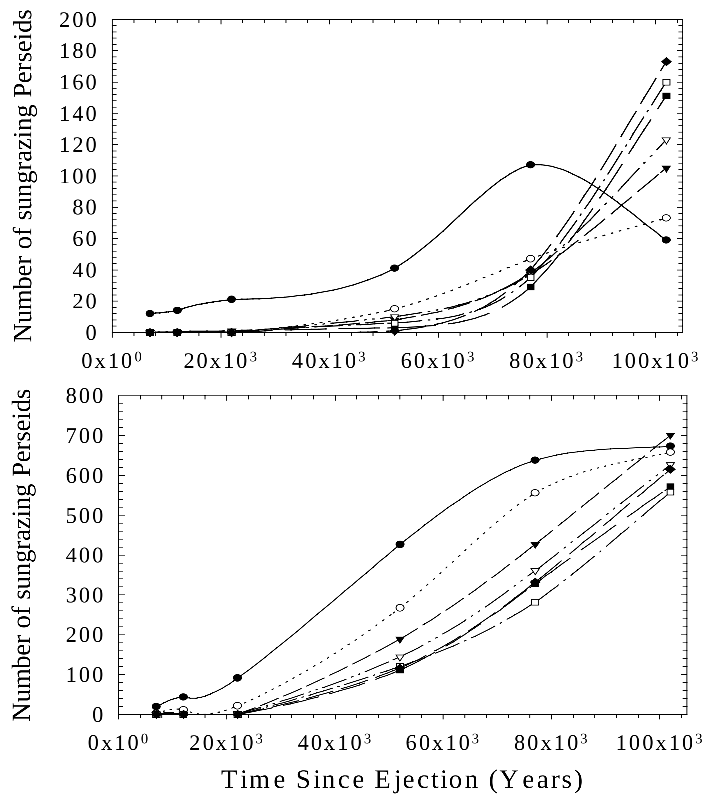

Fig 4.23: The number of Perseids that enter sungrazing states as a function of time since ejection for orbit #1 (top) and orbit #2 (bottom) for all seven mass categories. Symbols are the same as in Fig 4.21.

The removal rate resulting from entry into a sungrazing state is comparable to this value only for the largest meteoroids. The actual removal time is typically at least several times larger than this lower limit (depending on mass) based on our integrations, with some combinations of initial seed ejection orbit and masses showing loss rates which correspond to survival times almost two orders of magnitude longer than this lower limit.

From all of the above considerations, it is apparent that a Perseid meteoroid can, on average, survive for a minimum of several 10^5 years before being removed by one of these loss mechanisms, thus testifying to the possible great age of the stream, which we suspect is limited only by the capture time of Swift-Tuttle.

4.6 Future Activity of the Perseids

If the modelling results presented here are representative of the true Perseid stream, then some predictions of the time and strength of the activity of the stream for the next several years may serve to validate the model. In Table 4.8 is given the predictions of the peak time and strength for the outburst maximum for the Perseids from 1997-1999. The composition of each of these outburst maxima, in terms of the fraction of encountered meteoroids from the three most significant perihelion passages of Swift-Tuttle, summed over all models, is also presented. If the locations of maximum and levels of activity are found to be in good agreement with observations over the next few years, this will present the opportunity to record Perseid meteoroids whose ejection origin is somewhat constrained and for which precision observations would be most valuable as a result.

Over the longer term, Fig. 4.5 shows that the activity of the Perseids is expected to wax and wane and that the strength of the outburst maximum should be quite variable over the coming years. In particular, a minimum in annual activity from the outburst portion of the stream might be expected circa 2001-2 and a subsequent revival in 2004-2006. The latter increase in activity would be the direct result of the close approach to stream meteoroids by Jupiter early in 2003.

Year	Weighted Location of Maximum (J2000)	Contributing ejection epochs	Estimated ZHR of outburst maximum
1997	139.68 ° ± 0.04°	1479 (0.31) 1079 (0.17) 0826 (0.14)	95 ± 6
1998	139.73° ± 0.05°	1079 (0.20) 0826 (0.14) 1479 (0.11)	111 ± 6
1999	139.76 ° ± 0.05°	1079 (0.18) 0826 (0.16) 0698 (0.13)	115 ± 8

Table 4.8: The times of recent past and future theoretical locations for the peak times of the outburst portion of the Perseid stream and the approximate ZHRs (scaled to the mean average main peak ZHR of 86±1 found in Chapter 3).

4.7 Conclusions

From analyzing the results of the numerical modelling of the stream we may draw several conclusions pertinent to the opening questions presented in the introduction:

(1) The initial ejection conditions (which are typically of order several 10 - 100 m/s for visual-sized Perseids at perihelion for the models used here) play a central role in the final observed distribution of Perseid meteoroids at the Earth over time-scales of order ~5 cometary revolutions. After this interval, the effects of planetary perturbations and radiation forces begin to dominate the subsequent evolution of the stream, an effect manifested in the changing radiant size at present as a function of the time since ejection and by the lack of difference in the relative final activity as seen at Earth due to all the different ejection models from older ejections.

The choice of sun-centred cone angle makes only a marginal difference to the final activity outcomes. Different cone angles produce small changes to the total length of time over which activity occurs in any one year, particularly for recent ejections, with larger cone angles associated with longer activity. Narrower cones also limit the range of masses of Perseids subsequently accessible to Earth for more recent ejections.

From the model outputs, dust ejected at larger distances from the sun has a very minor effect on the final activity of the stream observed at Earth. The primary reason for this is the assumption of uniform ejection over the allowable range of true anomalies, which automatically concentrates the majority of the ejections close to perihelion. The outlying dust tends to end up on the periphery of the overall nodal longitude distributions (see Fig. 4.11).

The density (and thus the range of β) assumed for the meteoroids have the largest effect on the final distributions. That the evolutionary path is so sensitive to the assumed density of the particles is apparent by the systematic and consistent change in the number of meteoroids observed at Earth within each model as density is changed (cf Table 4.2). In particular, the number of meteoroids encountered increases with increasing assumed density (larger β). The change in density is related to both the ejection velocity and radiation pressure (both values increasing as density decreases for a given mass meteoroid). However, since 109P/Swift-Tuttle's descending node has been outside Earth's orbit for the last 2000 years, all meteoroids destined to encounter the Earth must be perturbed inward. Higher ejection velocities allow some meteoroids to have osculating orbits at ejection with lower nodal radii than the parent comet. One possibility is that this is a result of radiation pressure and differential perturbations moving the meteoroidal nodal points further out from the sun and of these forces being greatest for the lower density particles, as confirmed directly in 4.1. In this case, the effect dominates over the inward nodal motion caused by the initial ejection velocity dispersion. This effect is most noticeable on those large-β meteoroids that are ejected with large velocities along the comet's orbital motion and hence are in even lower energy orbits than the parent comet. Alternatively, the higher ejection velocities may simply spread the nodal "footprint" of the high-β meteoroids over a wider region than for lower-β and lead to lower concentrations everywhere (including near the Earth).

In using observations to constrain the model output, radiant location and orbital element distributions were found to be subject to measurement errors substantially larger than those intrinsic to the actual physical dispersions predicted by all models investigated. A quantitative assessment of the goodness of fit between the observed and predicted peak flux of the outburst portion of the stream and the location of the outbursts for the years 1989-1996 demonstrated that models 22 and 21 provided the best overall fit respectively. The lowest ejection velocity model (model 1 - distributed production) showed significantly poorer fits to the flux than did the other models for Swift-Tuttle. This suggests that very low ejection velocities of a few m/s to a maximum of a few tens of m/s are not representative of the decay process associated with Swift-Tuttle and that the density of meteoroids associated with the outburst portion of the stream is of the order 100-1000 kg m^{-3}.

At the other extreme, the very high ejection velocities recently proposed to explain the distribution in semi-major axes within the stream by Harris and Hughes (1995) and Williams (1996) are also not consistent with observation. In particular, by using such high ejection velocities (0.6 km/s near perihelion), it was found that the geocentric radiant dispersion from 1862 would be greater than 0.5 degrees. Our results (see Fig 4.6) suggest that “normal” ejection velocities from 1862 would produce radiant dispersions close to 0.1 degrees at present. From our simulations, the Perseid outbursts from 1991-1994 consist primarily of material ejected in 1862. Shiba *et al.* (1993) report photographic observations of the 1991 outburst showing a radiant dispersion of ~0.1 degrees from seven of nine photographed Perseids, while Spurny (1995) reports that the radiant dispersion for the 1993 outburst was 0.3 degrees for the concentrated portion (13 of 19 recorded Perseids) during that outburst. As individual radiant errors have not been incorporated into these measures, each of the observed dispersions represent upper limits with the true dispersions being smaller. As such, the ejection velocities we have employed appear to match the observed radiant sizes well and at the same time rule out the very high (~0.6 km/s at perihelion) ejection velocities proposed elsewhere. This is also consistent with our earlier remarks concerning the inadmissibility of orbital elements for the determination of original ejection velocities using current photographic techniques given the present size of their measurement errors.

(2 and 3) The location of the outburst portion of the Perseid stream has changed position over the last eight years due to a change in the age of the meteoroids found in this portion of the stream during that interval. From the simulation results, the outbursts from 1988-1990 were principally composed of meteoroids ejected in 1610 and 1737, while the 1991-1994 maxima consisted of material released in 1862 and 1610. The most recent outbursts (1995-1996) are from particles released in 1479 and 1079. The progressive relative increase in the solar longitude of the maxima in the years away from 1993 is due to the influence of the older ejections, released from the parent comet at larger nodal longitudes than the comet's current location and were further increased due to secular perturbations.

The high activity from the stream, particularly in the years 1991-1994, is due in part to the return of Swift-Tuttle and the numerous meteoroids in the Perseid stream with very similar periods to the parent comet. This, however, is a necessary but not sufficient condition for the occurrence of the outbursts. An impulsive change inwards of the nodal radius of the youngest portion of the stream due to a close approach to the stream orbit by Jupiter in 1991 was the additional condition sufficient to ensure that significantly enhanced activity from the shower occurred. This also explains the sudden onset in 1991; prior to this time meteoroids from 1862 were generally outside Earth's orbit and inaccessible to it as a result.

The discrepancy in the observed times of peak nearest Swift-Tuttle's perihelion passage (particularly in 1993 and 1994) could be due to a strong asymmetry in dust production during the 1862 passage of Swift-Tuttle. In particular, observations from that epoch indicate a strong tendency for ejections to have a large component of their total velocity in the positive normal direction relative to Swift-Tuttle's orbit. This tends to produce activity at Earth in the present epoch with larger nodal longitudes than the parent comet and may explain the difference between the (earlier) model predicted peak times and those observed nearest Swift-Tuttle's return when ejecta from 1862 predominated.

Our results also suggest that some smaller levels of "outburst" activity from the stream should have been visible well before the return of Swift-Tuttle as a result of the direct perturbations from Jupiter and Saturn. That no definitive visual observations of prior outbursts of the stream exist may be due to the fact that the first global synthesis of large numbers of visual observations of the stream did not occur until 1988. Thus the appearance of an early maximum in that year may not be intrinsic to the stream but only to the scrutiny with which it was observed. Indeed, Lindblad and Porubcan (1994) investigated the solar longitude distribution of previous photographically observed Perseids and concluded that the present outburst maximum was detectable as early as 1950. It is interesting to note further that on the basis of the present simulations we expect that some enhanced activity associated with the outburst portion of the stream should have been most apparent in the years around 1921, 1933, 1945, 1951, 1957, 1969, and 1980 with the maxima in 1921, 1945, 1957 and 1980 most prominent. Kronk (1988) lists the years 1920, 1931, 1945 and 1976-1983 as unusual for their reportedly high activity. Given the vagaries of moonlight and sparse observer distributions in these periods, there appears to be a remarkable concordance between the two lists. It is particularly noteworthy that several other studies of the 1980 Perseid return, in particular, suggest enhanced activity, such as that of Russell (1990) who suggested on the basis of his photographic observations that the 1980 Perseids may have been particularly prone to fragmentation and therefore of recent origin. Simek (1987) summarized nearly 30 years of radar observations of the Perseids and found that the 1980 return was the strongest recorded from 1958-1985 (with all the years from 1962-1972 having no observations), while Bel'kovich *et al.* (1995) determined that the returns from 1980-1982 were the strongest as recorded visually over the interval 1972-1990 from the former Soviet Union.

(4) From comparison of the radiant size of the Perseid stream and our model estimates of the change in radiant dispersion with age, the photographic-sized meteoroids in the main core of the stream are approximately 40 000 years old. Using the rate of change in the apparent location of the maximum, a lower limit of 11 000 years is obtained for the core of the stream. Similarly, using the width of the ZHR profile of the stream compared to the theoretical estimates yields another lower limit estimate for the central portion of the stream of 14 000 years. The photographic radiant locations at maximum are reproduced in the modelling with ejections 15-20 000 years of age. These estimates, along with their errors as given in Sect 4.4, are most consistent with a core population of Perseids having mean ages of order $(25 \pm 10) \times 10^3$ years. It is instructive to note that from the long-term integrations of the parent comet in Sect. 4.3, the most probable evolutionary paths for Swift-Tuttle all have nodal distances less than 0.1 A.U. from the Earth over the last 20 000 years; we would suggest that it is the dynamics of Swift-Tuttle's orbit over the last 20-30 millennia which control the highest activity portion of the stream presently visible at Earth.

The long duration of the Perseid shower indicates that the total age of the stream is much older. Our integrations show that some activity from the shower may be detectable at Earth for a significant portion of the entire year if the shower is as young as 10^5 years. The currently accepted duration of the shower of 40-45 days implies a lower limit for the age of the stream of order 10^5 years. It is not possible to be more precise given the uncertainties in the total length of time activity of the stream is visible at Earth and the precise evolutionary path followed by Swift-Tuttle.

A portion of our integrations suggests that, given enough time, some Perseid meteoroids may begin encountering the Earth at their ascending nodes in mid-March. Several candidate showers which are documented, but whose orbital elements are poorly known, have been identified. The existence of such a shower and positive association with Swift-Tuttle would imply a stream age of at least 50 000 - 75 000 years.

(5) We find that the current nodal progression rate of the stream (averaged over all models and masses for the last 2000 years) is $(2.2 \pm 0.2) \times 10^{-4}$ degrees/year. This is in good agreement with the observed rate of change of the location of the peak of the shower in historical times found by Hughes and Emerson (1982) of $(3.8 \pm 2.7) \times 10^{-4}$ degrees/annum.

(6) The Earth has a minor effect on the long-term evolution of the stream. In general terms we have found that the Earth contributes approximately ~10% to the total nodal dispersion of the stream and increases the radiant dispersion by a similar amount over time scales of order many thousands - tens of thousands of years. The Earth's scattering effect on the stream is also visible in the rms spreads in the orbital elements (a,i,ω), with the rms scatter becoming smaller in these elements when the Earth is removed. The effect is apparent, but far from dominant, in these orbital element dispersions with small number statistics becoming increasingly important for the oldest ejections. The Earth plays no perceptible role in moving Perseids into hyperbolic orbits, but may play some role in shepherding Perseids into sungrazing states.

(7) Two dynamical effects remove Perseids from the stream: hyperbolic ejection due to Jupiter (and to a lesser degree Saturn) and entry into sungrazing states. The relative importance and absolute amount of loss due to these mechanisms depends on the precise evolutionary path assumed for Swift-Tuttle and also varies by mass. The smallest Perseids tend to be preferentially removed first due to their lower average orbital energies. The rate of removal varied dramatically between the two assumed seed orbits (and by mass) with as many as 35% of the initial Perseid population hyperbolically ejected after 10^5 years for small meteoroids using seed orbit #2 while seed orbit #1 produced a loss rate of 1% over the same interval. Typically it required 40 000-80 000 years before any significant number (>0.1% of the initial population) was removed due to either of these two effects, but the actual number varied significantly from case to case.

(8) The delivery of Perseid meteoroids into Earth-intersecting orbits is principally controlled by the evolutionary path of the parent comet. The closest approach distance between the osculating orbit of Swift-Tuttle at the time of release of the meteoroids and the number of Perseids visible at the present time is strongly correlated over the last 2000 years. Over the longer term, the assumed starting orbit for the initial ejections critically influences the subsequent development and activity of the shower as seen from Earth. In the short term, impulsive perturbations due to Jupiter and Saturn control the magnitude of the outburst component of the stream and thus the amount of relatively “fresh” Perseid material visible at the Earth.

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