

66-200796  
copy 1

NOTICE TO READER: THE MATERIAL  
REPRODUCED IN THIS COPY MAY HAVE  
BEEN PROTECTED BY COPYRIGHT.  
FONDREN LIBRARY, RICE UNIVERSITY.

COLLISION PROBABILITIES WITH THE PLANETS AND THE  
DISTRIBUTION OF INTERPLANETARY MATTER.

By E. J. ÖPIK.  
(Armagh Observatory.)

(Communicated by E. M. Lindsay.)

[Read 8 MAY, 1950. Published 16 APRIL, 1951.]

SUMMARY.

FORMULAE for the probability of collision with a planet of a particle crossing the orbit of the planet are derived and applied to various objects. The secular advance of the perihelion with respect to the node renders collisions possible in all cases of a favourable distance range. The rôle of perturbations at close encounters is also considered.

In the solar system there are known at present 28 cases of objects crossing the orbit of only one planet ; these are Pluto with respect to Neptune, all the remaining being asteroids with respect to Mars. The expectation of life with respect to collisions for these objects ranges from  $2 \times 10^9$  to  $3 \times 10^{10}$  years.

All the other cases of crossings in the inner portions of the solar system are multiple crossings and refer to objects moving in orbits of large eccentricity but small inclination suggestive of results of close encounters, such as the short-period comets, some meteor showers, and the Apollo-Adonis group ; they are short-lived, of the order of  $10^8$  years, and can be only transient members of the inner portions of the solar system ; this group is called "zodiacal particles."

The actual stability of the Trojan group of planets against perturbations for time intervals of the order of the age of the solar system is proved by their mere existence, as otherwise they would have been destroyed by physical collisions with Jupiter in about  $10^6$  years.

The ecliptical portions of the zodiacal light may be chiefly due to the zodiacal particles ; an interstellar photometric component of the zodiacal light can be calculated from the absorption coefficient of galactic space without any other specific assumptions and is found close to the value determined photoelectrically at the McDonald Observatory for the pole of the ecliptic.

A planetoidal component of large particles ( $r > 100\text{cm}$ ) with small eccentricities and inclinations, unaffected by the Poynting-Robertson effect, is definitely absent or photometrically inconspicuous in the portions of the zodiacal light inside the orbit of Mars.

PROC. R.I.A., VOL. 54, SECT. A.

[25]

LIBRARY COPY

APR 10 1964

MANNED SPACECRAFT CENTER  
HOUSTON, TEXAS

Of the smaller particles belonging to the primordial aggregate of the solar system and moving in orbits of small eccentricity, the Poynting-Robertson effect combined with collisions, chiefly with Jupiter, would produce at present in the inner portions of the solar system a peculiar selection of particle sizes, those about  $r = 0.03$  to  $6$  cm being practically absent; the photometric properties of the  $F$ -corona as interpreted by Allen and van de Hulst might well be due to such primordial dust, about  $r = 10^{-3}$  cm, driven from outside toward the sun at such a rate that it passes the dangerous zone of Jupiter too fast to be caught by that planet. If this explanation holds, telescopic meteors of the 16-20 magnitudes should be extremely abundant.

A specific effect, due to non-uniform re-radiation of the absorbed energy by a rotating body in which a lag in the surface temperature with respect to insolation occurs, is considered; the effect is connected with the name of Yarkovsky who first published it at St. Petersburg half-a-century ago. Combined with the Poynting-Robertson effect, with probable speeds of rotation calculated from mutual collisions, the Yarkovsky effect proves important for directly rotating particles of  $r = 1$  to  $10$  cm which tend to accumulate in the portions of the solar system occupied by the inner planets and the zodiacal light.

1. *Zodiacal Light and Corona.* Allen (1) and van de Hulst (2) have shown that the  $F$ -component of the solar corona may be due to a diffractive scattering by small particles of the zodiacal light; the particles may be at any distance from the sun, the effect depending primarily upon the phase-angle and the size of the particle; it is the same effect which leads to the brightening of the sky background near the solar limb. Allen (1) finds that particles of a constant radius near  $10^{-3}$  cm, with a space-density inversely proportional to the distance from the sun, can account for the  $F$ -component when the space-density at 1 astr. unit is  $2 \times 10^{-23}$  g cm $^{-3}$ . Van de Hulst (2) assumes a frequency of the radii similar to the distribution of meteor sizes; he finds a space-density of  $5 \times 10^{-21}$  g cm $^{-3}$ , assumed constant and about  $10^4$  times more than can be inferred from visible meteors. The difference between the two estimates lies in their assumptions; diffractive effects are most efficient for small particles; assuming for all particles a most efficient size, Allen gets along with a minimum amount of matter, the postulated increase of the space-density toward the sun working in the same direction. However, the assumption of particles of a constant size is somewhat artificial and does not correspond to our direct knowledge of the distribution of the sizes of celestial bodies. Unless there are particular reasons for such an assumption (*cf.* below), a "meteoric" distribution of the sizes of the particles might appear to be more plausible; in such a case Allen's space-density should be increased by a factor of the order of 25. The remaining discrepancy may reflect the different assumptions as to the space-density—distance relation. Thus, both authors practically agree in ascribing the  $F$ -corona to scattering by small particles of the order of

$10^{-3}$  cm ; the discrepancy in space-density depends upon assumptions which have little connection with the optical phenomenon and cannot be proved or disproved by it.

In any case, the space-density of the zodiacal-light medium as inferred from the spectrophotometric observations of the corona would require either very much more meteors to be seen on the earth than actually observed or a considerably steeper decrement in meteor numbers with increasing size than revealed in the visual range. Van de Hulst (2) tries to evade this difficulty by assuming a quiescent zodiacal-light cloud with respect to the earth, in other words, a cloud of particles moving in orbits of small eccentricity more or less parallel to the motion of the earth ; however, such an assumption makes the disagreement even worse ; the smaller the original unperturbed relative velocity of the cloud, the larger will be the resulting intensity of a meteor shower produced from it by the condensing action of the earth's gravitational field. Consider first a pure two-body problem of particles subject only to the attraction of a planet of mass  $m$  and radius  $R$  ; the number of particles intercepted by the planet in unit of time is  $\pi\sigma^2\rho u$ , where  $\sigma$  is the effective target radius for capture,  $u$  the relative velocity and  $\rho$  the space-density "in infinity" ;  $s$  is the distance of the asymptote of the hyperbolic orbit from the centre of  $m$  when the periastron distance is  $R$ . The velocity in periastron is

$$v_0 = \sqrt{u^2 + s^2}, \quad (1)$$

where

$$s = \sqrt{\frac{2 G m}{R}} \quad (2)$$

is the velocity of escape from the surface of  $m$ .

Conservation of angular momentum requires  $\sigma u = R v_0$ , whence

$$\sigma^2 = R^2 \left( 1 + \frac{s^2}{u^2} \right) \quad (3)$$

and the intensity  $\nu$  of a meteor shower produced by the cloud and intercepted by  $m$  becomes  $(\pi\sigma^2\rho u)/(\pi R^2)$  or

$$\nu = \rho u \left( 1 + \frac{s^2}{u^2} \right). \quad (4)$$

This expression has a minimum at  $u = s$  and tends to infinity both at  $u \rightarrow \infty$  and  $u \rightarrow 0$ . Hence a quiescent cloud would require an infinite intensity of the meteor shower.

In the actual case of the earth the problem is complicated by the presence of a third body—the sun, of mass  $M$  ; nevertheless qualitatively the situation

remains the same. The two-body problem represents a plausible approximation well within a radius of action  $D_0$  around the earth (related to Roche's limit), inside which the acceleration of  $m$  upon the particle exceeds the perturbing (or tidal) force of  $M$ ; when  $D_0$  is small as compared with  $a_0$ , the mean distance of  $m$  from  $M$ , we may set

$$D_0 = a_0 (m/2M)^{\frac{1}{3}}. \quad (5)$$

In (4) and (3), for a cloud entering the sphere of action with a velocity  $u$ ,  $s$  is to be substituted by

$$s' = \sqrt{2Gm \left( \frac{1}{R} - \frac{1}{D_0} \right)}. \quad (6)$$

For the earth,  $D_0 = 0.01149$  a.u. =  $270 R$ , whence

$$s'^2 = 0.9963 s^2,$$

or so small a difference that the formulae of the two-body case may be applied without change.

The minimum velocity of impact is  $s = 11.1$  km/sec for the earth, sufficient to render the meteors visible; the average relative velocity may be around  $u = 6$  km/sec, corresponding to asteroidal eccentricities and inclinations; according to (3) this gives  $\sigma^2/R^2 = 4.5$ , by which factor the van de Hulst discrepancy is to be augmented.

2. *Sweeping Action of the Earth.* It may seem tempting to assume the absence or scarcity of large meteoric particles in the zodiacal-light cloud, in order to explain the actual failure to observe large numbers of such meteors; in the case of a cometary or an interstellar origin of these particles such an assumption becomes a necessity indeed. However, let us first examine the assumption that the zodiacal light particles move around the sun in orbits of small eccentricity. In such a case the difficulty viewed by van de Hulst disappears from quite different reasons. The failure to observe large zodiacal-light meteors is then simply explained by their absence from our vicinity: the earth has swept clean all the space around its orbit, absorbing in the course of time practically all particles which persistently remained in its vicinity (the bearing of the Poynting-Robertson effect on the persistence of small particles is considered in a later section). The effective volume swept by the earth within a time  $t$  is

$$V = \pi \sigma^2 u t. \quad (7)$$

With  $u = 6$  km/sec,  $\sigma^2 = 4.5 R^2 = 2 \times 10^8$  km<sup>2</sup> and an age of the earth  $t = 10^{17}$  sec we get  $V = 4 \times 10^{26}$  km<sup>3</sup>, equal to the volume of a sphere of

3.1 astr. units radius ; the sweeping action being confined to a much smaller volume in the vicinity of the earth's orbit, this space must have been cleaned of any debris moving in orbits of small eccentricity. Survive only such small particles which, on account of the Poynting-Robertson effect, remain in the vicinity of a planet for too short a time, and such large particles which have little chance to get into the path of the planet, thus particles which move between the planets in orbits of relatively small eccentricity.

3. *Probability of Collision with a Planet.* Consider a planet in a conventionally circular orbit of radius  $a_0$ , and a particle with orbital elements  $a, e, i, \omega$ , the inclination and node being referred to the plane of the planetary orbit. A collision may occur only when  $a(1 + e) > a_0 - \sigma$  or  $a(1 - e) < a_0 + \sigma$ . When  $i$  is not negligible, a collision is possible only for a limited range of the true anomaly in the node, which means definitely prescribed limits of  $\omega$ , the perihelion distance from node ; thus, for a given epoch collisions may not be possible. However, since the time of Laplace and Lagrange it is known that secular perturbations lead to progressive changes in the longitudes of the node (mostly decreasing) and of the perihelion (mostly increasing), so that their difference  $\omega$  undergoes also a progressive change (mostly increasing) (3). As referred to the ecliptic of 1850, the inner planets yield following periods for  $\omega$  completing one revolution :

	Mercury	Venus	Earth	Mars
Period, years	133000	120000	64000	49000

In our case we are concerned with the motion of the perihelion of the particle referred to the variable orbital plane of the planet. A particle near the orbit of the earth will undergo secular perturbations about the same as the earth *plus* perturbations by the earth itself which, because of the closeness of the orbits, may exceed the perturbations by all the other planets together (the secular perturbations of the earth upon Venus are already of the same order of magnitude as those of Jupiter) ; in other words, an interplanetary particle is expected to show, with respect to its node in the orbital plane of a neighbouring planet, a faster secular variation of  $\omega$  than the secular variation of the planet itself with respect to a fixed frame of reference. Whatever the actual irregularities in the variation of  $\omega$ , for time intervals of the order of  $10^6$  to  $10^8$  years, such as the life-time of a particle may be (*cf.* below), we may assume these irregularities to be distributed at random ; with a period of  $\omega$  of  $10^4 - 10^5$  years and less, or much shorter than the life-time of the particle, the probability for the occurrence of  $\omega$  within definite limits  $\omega$  and  $\omega + \Delta\omega$  is then  $\Delta\omega/2\pi$ . If  $\Delta\omega$  covers the range of  $\omega$  within which a collision can occur, the average probability of collision per revolution of the particle may be written as

$$P = \frac{2\Delta\omega}{\pi} P'. \quad (8)$$

Here  $P'$  is the average probability of collision during a passage through the node within the critical range from  $\omega$  to  $\omega + \Delta\omega$ ; a factor of 4 is included, to allow for the two nodes and the two symmetrical vectors of the particle's orbit.

With  $\sigma/a_0$  small,  $i$  and  $e$  not too small, the trajectories of the planet and the particle (if undisturbed) may be considered as rectilinear near the point of approach, i.e., near the node; thus first-order differential formulae may be used to determine  $\Delta\omega$  and  $P'$ . For the sake of simplicity we assume  $a_0$  as unit of distance and  $W$ , the orbital velocity of the planet, as unit of velocity; we set

$$A = a/a_0, \quad U = u/W, \quad S = s/W, \quad \tau = \sigma/a_0, \quad Q = R/a_0.$$

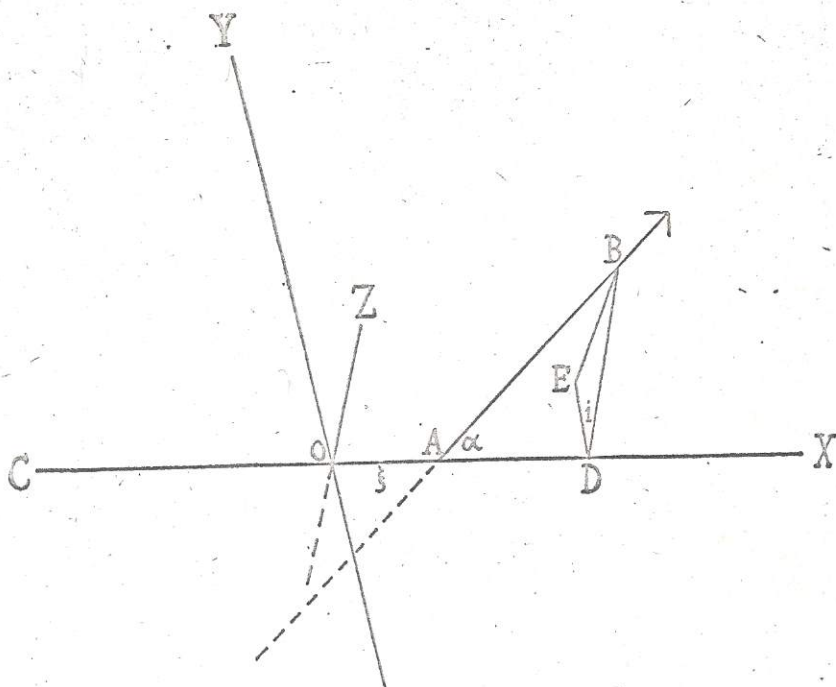


FIG. 1.

Let rectangular coordinates be chosen (Fig. 1) such that OY is the trajectory of the planet, OX the line of nodes (or CO = 1 the vector from the sun), OZ at right angles to the plane of the planet's orbit, and AB the trajectory of the particle intersecting the node line at a distance OA =  $\xi$  from the origin. The coordinates of the particle at B be OD =  $x$ , ED =  $y$ , BE =  $z$ . The distance of B from OY is

$$d^2 = x^2 + z^2.$$

From the triangles BED and BAD, with  $AD = x - \xi$  and  $\angle BAD = \alpha$  (angle between tangent and radius-vector of the particle's orbit in the node), we have

$$z = (x - \xi) \operatorname{tg} \alpha \sin i,$$

whence

$$d^2 = x^2 + (x - \xi)^2 \operatorname{tg}^2 \alpha \sin^2 i. \quad (9)$$

The closest approach between the trajectories of the particle and the planet corresponds to a minimum in  $d^2$ ,  $\frac{\partial (d^2)}{\partial x} = 0$ ; which yields

$$x = \xi / (1 + \operatorname{ctg}^2 \alpha \operatorname{cosec}^2 i) \quad \text{and} \quad d_{\min}^2 = \xi^2 / (1 + \operatorname{ctg}^2 \alpha \operatorname{cosec}^2 i). \quad (10)$$

By setting  $d_{\min} = \tau$ , the target radius for collision, we get the limiting values of  $\xi$  between which collisions may occur at all as

$$\xi_0 = \pm \tau \sqrt{1 + \operatorname{ctg}^2 \alpha \operatorname{cosec}^2 i}. \quad (11)$$

In the node the true anomaly of the particle is  $\omega$ ; the polar equation of the ellipse yields for the radius vector in the node

$$1 + \xi = \frac{A(1 - e^2)}{1 + e \cos \omega}. \quad (12)$$

Let  $\xi = 0$  at  $\omega = \omega_0$ , and let  $\omega = \omega_0 + \delta\omega$ ,  $\xi = \delta r$ ;  $\delta r$  and  $\delta\omega$  being small, the differential formula  $r\delta\omega = \delta r \operatorname{tg} \alpha$  applies, or, at  $r = 1$ ,

$$\delta\omega = \xi \operatorname{tg} \alpha. \quad (13)$$

Hence the range in  $\omega$  within which collisions are possible, corresponding to the range in  $\xi$  from  $-\xi_0$  to  $+\xi_0$  or to  $2|\xi_0|$ , according to (11) and (13) becomes

$$\Delta\omega = 2\tau \sqrt{\operatorname{tg}^2 \alpha + \operatorname{cosec}^2 i}. \quad (14)$$

Differentiation of (12) yields the formula for  $a$  at  $r = 1$  as

$$\operatorname{ctg}^2 \alpha = \left(\frac{1}{q} - 1\right) \left(1 - \frac{1}{q'}\right), \quad (15)$$

where  $q = A(1 - e)$ ,  $q' = A(1 + e)$ .

$$\Delta\omega = \frac{2\tau}{U_x \sin i} \sqrt{U_x^2 + U_z^2}$$



$$P' = \frac{\pi}{4} P_0, \quad (16)$$

where  $P_0$  is the probability of collision for a central passage MN.

For a central passage, let at time  $t = 0$  the particle be in O, the planet at  $y = OP = \eta$ . If  $U_x, U_y, U_z$  are the components of the relative velocity of the particle with respect to the planet,  $U^2 = U_x^2 + U_y^2 + U_z^2$ , the distance between particle and planet at a moment  $t$  during the central passage (if undisturbed) is given by

$$\begin{aligned} \Delta^2 &= (U_x t)^2 + (\eta - U_y t)^2 + (U_z t)^2, \quad \text{or} \\ \Delta^2 &= \eta^2 - 2\eta U_y t + U^2 t^2. \end{aligned} \quad (17)$$

The minimum distance corresponds to  $\frac{\partial(\Delta^2)}{\partial t} = 0$ , which gives  $t = \eta U_y / U^2$ ; the minimum distance becomes

$$\Delta^2_{\min} = \eta^2 \left( 1 - \frac{U_y^2}{U^2} \right).$$

Equalling this to  $\tau^2$ , we get the extreme range in the position occupied by the planet at  $t = 0$  for a successful collision as follows:

$$\eta_0 = \pm \frac{\tau U}{\sqrt{U_x^2 + U_z^2}}. \quad (18)$$

The probability of collision at central passage is then obviously equal to the probability of the planet to be found between  $y = \pm |\eta_0|$ , or to

$$P_0 = \frac{|\eta_0|}{\pi}. \quad (19)$$

Applying Kepler's second law, the velocity components in the units chosen are determined as follows:

$$\left. \begin{aligned} U_x &= \pm \sqrt{A(1-e^2)} \operatorname{ctg} \alpha; \\ U_y &= + \sqrt{A(1-e^2)} \cos i - 1; \\ U_z &= \pm \sqrt{A(1-e^2)} \sin i, \end{aligned} \right\} \quad \begin{array}{l} v_r = \sqrt{2 - \frac{1}{a}} \cos \alpha \\ (20) \end{array}$$

The total relative velocity is

$$U^2 = 3 - \frac{1}{A} - 2 \sqrt{A(1-e^2)} \cos i. \quad (21)$$

From (20) we find

$$U_x^2 + U_z^2 = A(1 - e^2)(\cotg^2 \alpha + \sin^2 i). \quad (22)$$

Equations (22), (18), (19), (16), (14) and (8), after consecutive substitution yield (use being made of the expression for  $U_x$  in (20))

$$P = \frac{\tau^2 U}{\pi \sin i \cdot |U_x|} \quad (23)$$

as the probability of collision *per revolution* of the particle. The life-time in years is evidently

$$T = a^{3/2} / P, \quad (24)$$

when  $a$  is given in astronomical units; this is the time for which the probability of survival is  $1/e$ .

For calculation according to (23)  $U$  is given by (21); with (15) and (20) we find a convenient expression for the radial component of the relative velocity in the node at encounter as

$$|U_x| = \left[ \frac{(R - r)(q - e)}{Ra} \right]^{1/2}$$

$$|U_x| = \sqrt{2 - \frac{1}{A} - A(1 - e^2)}. \quad (25)$$

The orbital velocity of the planet, taken as unit, is  $\sqrt{\frac{GM}{a_0}}$ ; according to (2) the velocity of escape in our relative units then becomes

$$S = \sqrt{\frac{2m}{MQ}} \quad (26)$$

and the target radius, according to (3),

$$\tau = Q \sqrt{1 + \frac{2m}{MQU^2}}. \quad (27)$$

The use of the formula (23) is restricted to the validity of our first-order approximation; at very small values of  $i$  and  $|U_x|$  it fails. The limits of reliability may be assumed those for which the arcs  $\frac{1}{2}\Delta\omega$  and  $\eta_0$  can be substituted by their tangents: for a 10 per cent. accuracy these arcs should not subtend over 0.4 radians; further  $\tau < D_0/a_0$  or the target radius must not exceed the sphere of action. The last condition sets a lower limit to the relative velocity  $U$ ; when  $U$  is small, the unit under the square root of (27) can be neglected; from (27) and (5) the lower limit then obtains as

$$U > 2^{5/6} \left( \frac{m}{M} \right)^{1/6} Q^{1/2}. \quad (28)$$

For the earth this gives  $U > 0.0014$ , in units of its orbital velocity, as the limit of validity of our formulae; the limit is very low and of no practical significance. For small  $e$  and  $i$  the validity of the linear approximation may be considered in the specific case of  $A = 1$ . In this case formula (21) in first-order terms of  $e^2$  and  $i^2$  is reduced to

$$U^2 = e^2 + i^2; \quad (a)$$

also, from (25),  $|U_x| = e$ , from (20) (for small  $e$  and  $i$ ),  $|U_z| = i$ , from (27) (neglecting the unit) and (a),  $\tau^2 = \frac{2mQ}{M(e^2 + i^2)}$ , and from (15),  $ctg^2 a = e^2$ ; hence the limit of validity of the linear approximation requires: from (18),

$$|\gamma_0| = \sqrt{\frac{2m}{M} \frac{Q}{(e^2 + i^2)}} < 0.4, \quad (b)$$

and from (14),

$$\left| \frac{\Delta\omega}{2} \right| = \frac{1}{ei} \sqrt{\frac{2mQ}{M}} < 0.4. \quad (c)$$

Of these, (c) is the more restrictive one and yields for the earth  $e > 0.0063$ ,  $i > 0.0064$  or  $22'$  of arc. (c').

This covers also condition (28). For the other three inner planets (c) is even more favourable. The secular variations in  $e$  and  $i$  exceed the "margin of sharpness," (c'), by at least an order of magnitude (4); therefore, over long intervals of time the average eccentricity and relative inclination of the orbit of the particle, to be used in the above formulae, will always considerably exceed the margin (c'); thus, in the case of our solar system, the linear approximation introduced above will always be valid if due allowance is made for the average unsharpness of  $e$  and  $i$  produced by the secular variations of these elements.

4. *Corrections for Secular Variations and for the Eccentricity of the Planet's Orbit.* The above formulae can be directly applied when the eccentricity and inclination of the particle are large as compared with the amplitude of their secular variations, and when the eccentricity of the particle considerably exceeds the eccentricity of the planet's orbit; the same holds also in an opposite case, when the planet's eccentricity considerably exceeds  $e$  for the particle; in which case in formulae (21), (25) and (24)  $1/A$  is to be set for  $A$ , the planet's eccentricity for  $e$  and  $a_0$  for  $a$ .

In other cases corrections are required, to eliminate an overestimate of the probability of collision by assuming values of  $i$  and  $e$  which are small at the moment but cannot remain small over long periods of time. Safeguards against such a possibility need not be very precise. The following procedure may be adopted.

Allowing for the amplitude of secular variations in the  $ae$  and  $i$  vectors, in the case when  $e$  and  $i$  happen to be very small at the moment, a minimum average eccentricity of 0.02 and a minimum average  $\sin i = 0.02$  with respect to the invariable plane is assumed. The relative inclination to be used in formula (23) is to be defined as the statistical average for a random distribution of the nodes,

$$\sin i = \sqrt{(\sin i_1)^2 + (\sin i_0)^2}, \quad (29)$$

where  $\sin i_1$  and  $\sin i_0$  are the assumed averages for particle and planet referred to the invariable plane of the solar system.

As to the eccentricity of the planet's orbit, instead of determining its influence by a direct more complicated procedure, we retain form. (23) with the understanding that  $U$  and  $|U_x|$  are statistical averages of the relative total and the relative radial velocities of particle and planet, for a random relative distribution of the perihelia; thus  $U_x^2 = \overline{U_{1x}^2} + \overline{U_{0x}^2}$ ,  $U_{1x}$  referring to the particle,  $U_{0x}$  to planet. For a small eccentricity  $e_0$  of the planet the average is  $|\overline{U_{0x}}| = \frac{2}{\pi} e_0 (1 - e_0^2)$  (averaged by the anomaly, with  $A = 1$  in form. (20) and (15)) or practically  $\frac{2}{3} e_0$ . Hence the corrected formulae are

$$U_x^2 = 2 - \frac{1}{A} - A(1 - e^2) + \frac{4}{9} e_0^2, \quad (25')$$

and

$$U^2 = 3 - \frac{1}{A} - 2\sqrt{A(1 - e^2)} \cos i + \frac{4}{9} e_0^2, \quad (21')$$

valid for the case  $e > e_0$ , and when  $A(1 + e) > 1 + e_0$ ,  $A(1 - e) < 1 - e_0$ , i.e. when the two orbits can intersect in any point of the planet's orbit. When the last two inequalities are not fulfilled, e.g. when

$$A(1 + e) > 1 + e_0 > A(1 - e) > 1,$$

only an "overlapping fraction"  $f$  (reckoned by time, i.e. by area) of the planet's orbit around the aphelion can be approached by the particle; in this case a rough approximation can be used, by applying formulae (21) and (25) with  $A' = a/a_0(1 + e)$  for  $A$  and multiplying the resulting probability by  $f$ . Similarly, when  $1 > A(1 + e) > 1 - e_0$ ,  $A'' = a/a_0(1 - e)$  for  $A$  is to be used; when  $A(1 + e) > 1 + e_0 > 1 > A(1 - e) > 1 - e_0$  or  $1 + e_0 > A(1 + e) > 1 > 1 - e_0 > A(1 - e)$ ,  $A = a/a_0$  in equ. (25') and (21') is retained, with an appropriate value of  $f$ . In the numerical applications below, these cases are denoted as "partial crossings" (e.g. Adonis-Mercury); as the error in  $T$  hardly exceeds 10 per cent., it is not worth while using a more strict procedure.

5. *Close Encounters and Radiation Pressure.* Before a physical collision happens, close encounters may occur changing the orbital elements of the particle (5, 6) and thus the probability of collision. Of importance are not only the large changes; numerous small repeated perturbations are summing up according to the law of chance (13, 6). A close approach to one of the planets of the solar system is practically equivalent to an elastic collision, and even in the case of the large sphere of action of Jupiter the model works well. In the close approach the relative velocity  $U$  remains unchanged, whereas a redistribution of its components takes place. From approaches to only one planet (its orbit again being assumed as circular) and when  $U$  is small, only limited changes in the orbital elements and in  $P$  can take place: the particle is doomed to return to the spot of the previous encounter and, with the secular variations steadily working their way, is sooner or later captured by the planet. In the case of small  $U$ , i.e. when the orbits of the particle and the planet run close together, the probability of physical collision determines actually the life-time of the particle. When  $U$  is large, considerable changes in the orbital elements may occur; when  $U > \sqrt{2} - 1$ , a hyperbolic orbit may result under special circumstances of the encounter, the particle being thrown out of the solar system; but when  $U$  is below the above limit but large enough to produce eventually a considerable eccentricity so that another planet may be met, close encounters with the second planet will produce changes in the orbital elements of the particle, such that  $U$  no longer will remain constant with respect to the first planet, and *vice versa*; the changes of  $U$  will accumulate at random, no limit being set, and this "playing ball" with the particle may result finally in the particle being thrown out of the solar system, in falling into the sun, or at least in its orbital elements being changed so much that it no longer will contribute essentially to the space-density of its former surroundings. Thus, close encounters will contribute to the probability of the disappearance of the particle; in such a case the above probability of physical collision alone represents actually a minimum estimate for the probability of disappearance.

We retain the notations and units of Section 3 with the understanding that  $Q$ ,  $S$  and  $\tau$  refer to the closest approach of the particle in its passage near the planet, so that  $Q$  is the periastron (perigee) distance, not necessarily the physical radius of the planet; with the aid of (20), (21), (25) and (27) and from a consideration of the hyperbolic relative motion of the particle we have:

$$\operatorname{tg}\left(45^\circ + \frac{\psi}{4}\right) = \sqrt{1 + \frac{S^2}{U^2}}, \quad (30)$$

where  $\psi$  is the angle between the asymptotical incoming and outgoing directions.  $S$  is determined either by (26), or through the target radius:

$$\frac{S^2}{U^2} = 2 \cdot \left(\frac{m}{M\tau U^2}\right)^2 \left[1 + \sqrt{1 + \left(\frac{M\tau U^2}{m}\right)^2}\right]. \quad (31)$$

Consider the case of repeated approaches to only one planet (with a circular orbit) so that  $U$  remains constant, restricted by  $U < \sqrt{2} - 1$ ; (30) determines the change of direction after one encounter; but even when  $\psi$  is small, repeated encounters will sum up at random, making all relative directions possible. If the probability is reckoned per encounter, after a large number of encounters all directions of the vector  $U$  become equally probable, in which case the probability for a velocity component to range from  $U_x$  to  $U_x + dU_x$  is  $dU_x/2U$ , and the probability for a velocity vector to lie between  $U_x$  and  $U_x + dU_x$ ,  $U_y$  and  $U_y + dU_y$  is

$$p dU_x dU_y = \frac{dU_x dU_y}{4\pi U \cdot |U|}. \quad (32)$$

The velocity components determine the orbital elements of the particle. From (20) and (25) we have

$$\operatorname{tg} i = \frac{U_z}{1 + U_y}, \quad (33)$$

and 
$$A(1 - e^2) = (1 + U_y)^2 + U_z^2 = 2 - \frac{1}{A} - U_x^2. \quad (34)$$

Hence 
$$\frac{1}{A} = 1 - F, \quad (35)$$

and 
$$e^2 = F^2 + U_x^2(1 - F), \quad (36)$$

where 
$$F = 2U_y + U^2. \quad (37)$$

The perihelion and aphelion distances become

$$q = A(1 - e) = \frac{1 - \sqrt{F^2 + U_x^2(1 - F)}}{1 - F}, \quad (38)$$

$$q' = A(1 + e) = \frac{1 + \sqrt{F^2 + U_x^2(1 - F)}}{1 - F}. \quad (39)$$

From (35) and (37) obviously

$$1 - 2U - U^2 < \frac{1}{A} < 1 + 2U - U^2, \quad (40)$$

which yields the extreme limits of  $A$  when  $U_y = \pm U$ .  $A = \infty$  can be attained only when  $1 - 2U - U^2 \leq 0$ ,  $U = \geq \sqrt{2} - 1$  (the parabolic limit).

The eccentricity depends upon two independent parameters,  $U_x$  and  $U_y$ ; from (36),  $\frac{\partial(e^2)}{\partial(U_x^2)} = 1 - F > 0$  when  $A > 0$  (cf. (35), elliptical orbits); hence, at a given  $U_y$  the largest value of  $e^2$  (not the analytical maximum)

corresponds to the largest possible value of  $U_x^2$  which is  $U^2 - U_y^2$  (when  $U_z = 0$ ); with this in (36), the largest possible  $e^2$  at given  $U_y$  is

$$e^2 = 2U_y^3 + U_y^2(U^2 + 3) + 2U^2U_y + U^2;$$

this yields a maximum at

$$U_y = -\frac{1}{6} [(U^2 + 3) - \sqrt{(U^2 + 3)^2 - 12U^2}] = -1,$$

a minimum at

$$U_y = -\frac{1}{6} [(U^2 + 3) + \sqrt{(U^2 + 3)^2 - 12U^2}] = -\frac{U^2}{3},$$

an absolutely largest value for positive  $U_y$  at  $U_y = +U$  and another largest value for negative  $U_y$  at  $U_y = -U$  (not maxima, however). The first (maximum) value, for small  $U$ , is equal to  $-1$  and cannot be realized when  $U < 1$ ; there remain the two largest values of  $e$  at  $U_y = \pm U$ ; these occur exactly when  $A$  also attains its largest and smallest values. It follows that the largest aphelion distance, attainable through close encounters at  $U = \text{const.}$ , is given by (39) setting  $U_y = +U$ ,  $F = U^2 + 2U$ , and the smallest perihelion distance—by (38) with  $U_y = -U$ ,  $F = U^2 - 2U$ ; in these cases  $U_x = 0$ ,  $e = |F|$ ,

$$q'_{\max} = \frac{(1 + U)^2}{1 - 2U - U^2}, \quad q'_{\min} = \frac{(1 - U)^2}{1 + 2U - U^2}.$$

Table I contains these data. It may be noted that these extreme cases require  $U_z = 0$  or, according to (33),  $i = 0$ : if large eccentricities are produced by close encounters, the resulting inclinations will show a tendency to remain small.

TABLE I.

U	Largest Aphelion $U_y = +U, q = 1$			Smallest Perihelion $U_y = -U, q' = 1$		
	e	A	q'	e	A	q
0	0	1.000	1.000	0	1.000	1.000
0.05	0.1025	1.114	1.228	0.0975	0.911	0.822
0.10	0.2100	1.266	1.531	0.1900	0.840	0.681
0.15	0.3225	1.476	1.952	0.2775	0.783	0.566
0.20	0.4400	1.786	2.571	0.3600	0.735	0.471
0.25	0.5625	2.286	3.572	0.4475	0.691	0.382
0.30	0.6900	3.226	5.452	0.5100	0.662	0.325
0.35	0.8225	5.634	10.268	0.5775	0.634	0.268
0.40	0.9800	25.000	49.00	0.6400	0.610	0.220
0.4142	1.0000	$\infty$	$\infty$	0.6568	0.604	0.207
0.50	> 1	Hyperc.		0.7500	0.571	0.143
0.60	"	"		0.8400	0.543	0.087
0.80	"	"		0.9600	0.510	0.020
1.00	"	"		1.0000	0.500	0.000

In connection with the above, the problem of the largest possible expectation

of life at  $U = \text{const.} < \sqrt{2} - 1$  presents some interest. With the aid of (20), (25) and (35), (24) can be represented as

$$T = \frac{a_0^{3/2}}{r^2 U} \gamma, \quad (24')$$

where

$$\gamma^2 = \frac{U_x^2 (U^2 - U_x^2 - U_y^2)}{(1 - 2U_y - U^2)^3 (1 + 2U_y + U^2 - U_x^2)}. \quad (41)$$

$T$  attains a maximum when

$$\frac{\partial(\gamma^2)}{\partial(U_x^2)} = 0 \quad \text{and} \quad \frac{\partial(\gamma^2)}{\partial U_y} = 0.$$

The first condition yields

$$U_x^2 = (1 + 2U_y + U^2) - \sqrt{1 + 2U_y + U^2} (1 + U_y), \quad (42)$$

whence

$$\gamma_{\max}^2 = \text{Max.} \frac{[\sqrt{1 + 2U_y + U^2} - (1 + U_y)] (\sqrt{1 + 2U_y + U^2} - 1)}{(1 - 2U_y - U^2)^3}. \quad (43)$$

At  $U \rightarrow 0$ , the maximum takes place at

$$U_y \rightarrow + \frac{U}{\sqrt{3}} \quad \text{and} \quad \gamma_{\max}^2 \rightarrow \frac{\sqrt{3}}{9} U^3.$$

For  $U$  different from zero the analytical solution of (43) is too complicated; a numerical solution was made instead. It is convenient to represent

$$\gamma_{\max} = B U^{3/2}, \quad (44)$$

whence

$$T_{\max} = \frac{B \sqrt{a_0^3} U}{r^2} \text{ (years)}. \quad (45)$$

Table II contains the calculated values of  $B$ .

TABLE II.

$U$	0	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.4142
$U_y/U \text{ at max.}$	0.577	0.595	0.618	0.653	0.695	0.751	0.818	0.893	0.976	1.000
$B$	0.439	0.480	0.537	0.615	0.726	0.906	1.242	2.098	9.04	$\infty$

Formula (45) gives an upper limit to the life-time of a particle at  $U = \text{const.}$ ; this upper limit will never be realized (except when infinite) because, even when the orbital elements of the particle once assume the required "safest" set of values, the following encounters will change them and  $T$  will drop below the maximum value.

Radiation pressure had been neglected in the above formulae which refer to large particles. Although in the solar system radiation pressure becomes important only for  $r < 10^{-4}$  cm, for the sake of completeness some formulae for this case may be given. Let  $1 - \beta$  be the average ratio of radiation pressure to gravitation with respect to the sun (with respect to a planet it is always negligible). Then, in (20)  $\beta$  is to be inserted under the square root, whereas (15), (23) and (27) remain unchanged, with

$$|U_x| = \sqrt{\beta} \sqrt{2 - \frac{1}{A} - A(1 - e^2)}, \quad (25\beta)$$

$$U^2 = (1 + 2\beta) - \frac{\beta}{A} - 2\sqrt{\beta A(1 - e^2)} \cos i \quad (22\beta)$$

and

$$T = a^{3/2} / (P\beta^{1/2}). \quad (24\beta)$$

6. *Numerical Applications.* Table III contains data for the calculation of the collision probabilities with respect to the planets.  $\sin i_0$  refers to the invariable plane; this and  $e_0$  are estimated averages, or minimum values of these averages over long periods of time.

(a) *Crossings with a single planet.* Table IV contains all such known cases. The asteroids are from the 1948 list (?); noticeable is the crowding toward the latest numbers which indicates a peculiar observational selection; from the same circumstance it is obvious that the list is far from being complete. The diameter is estimated from

$$\log d = 3.63 + \log a(a - 1) - 0.2 m_0, \quad (46)$$

TABLE III.

*Collision constants for the planets.*

Planet	Mercury	Venus	Earth	Mars	Jupiter	Saturn	Uranus	Neptune
$a_0$	.. 0.387	0.723	1.000	1.524	5.20	9.54	19.19	30.07
$\sin i_0$	.. 0.14	0.07	0.03	0.05	0.02	0.03	0.02	0.02
$e_0$	.. 0.20	0.02	0.02	0.10	0.05	0.06	0.05	0.01
$Q, 10^{-5}$ units	4.18	5.63	4.26	1.48	8.90	4.07	0.866	0.589
$2m/MQ$	.. 0.00799	0.0870	0.1423	0.0433	21.5	14.1	10.1	17.2

where  $m_0$  is the magnitude at distance  $a$  from the sun and  $(a - 1)$  from the earth; a reflecting power equal to that of the Moon is assumed.  $U$  is the relative velocity, in units of the circular velocity at the assumed point of encounter;

$$T = \frac{\pi \sin i \cdot |U_x| a^{3/2}}{r^2 U \cdot f}$$

is the life-time for collisions;

$$f = \frac{1}{\pi} \arccos \frac{A(1-e) - (1-e_0^2)}{e_0 A(1-e)} \quad (47)$$

is the conventionally assumed overlapping fraction for partial crossings of the perihelion type, i.e. when  $A(1-e) > 1 - e_0$ .

Significant are the large values of  $T$  which only in two cases (Eros and Atlantis) fall below  $3 \times 10^9$  years; all these objects are long-lived as compared with the presumed age of the solar system; at least partly this may be the result of survival by selection, favouring the long-lived cases.

For an interval of time  $t$  the chances of a collision are

$$P_i = 1 - e^{-t/T}, \quad (48)$$

which with  $t = 3 \times 10^9$  yields quite considerable figures: 0.25 for Pluto, 0.73 for Eros, 0.11 for Aethra. For  $t = 10^8$ , the 27 asteroids of the list yield a combined expectation of collision with Mars  $\sum P_i = 0.345$ , or one collision

TABLE IV.

Single crossings. Pluto with respect to Neptune, all the rest with respect to Mars.

		132	323	391	433	475	699
Object	Pluto	Aethra	Brucia	Ingeborg	Eros	Oello	Hela
$d$ , km	..	89	54	30	20	35	23
$U$	..	0.512	0.470	0.447	0.289	0.444	0.392
$T$ , $10^9$ years	10.3	25.6	23.4	16.6	2.32	22.1	19.8
$f$	..	0.262	0.102	0.333	1.000	0.352	0.429
		719	887	985	1009	1011	1036
Object	Albert	Alinda	Rosina	Sirene	Laodamia	Ganymed	Porzia
$d$ , km	..	5.2	6.2	14	5.2	10	59
$U$	..	0.502	0.510	0.210	0.437	0.266	0.661
$T$ , $10^9$ years	7.92	6.92	5.68	11.6	5.92	16.7	4.50
$f$	..	1.000	1.000	0.284	0.725	0.406	1.000
		1134	1139	1170	1193	1204	1221
Object	Kepler	Atami	Siva	Atlantis	Renzia	Amor	Schorria
$d$ , km	..	6.6	12	22	5.2	13	1.8
$U$	..	0.442	0.298	0.479	0.228	0.205	0.483
$T$ , $10^9$ years	12.0	4.99	19.9	2.69	4.89	5.62	11.1
$f$	..	0.716	0.659	0.233	0.554	0.289	1.000
		1293	1310	1316	1374	1468	1474
Object	Sonja	Villigeria	Kasan	Isora	1933 PA	1935 QY	1938 UO
$d$ , km	..	12	23	11	11	26	10
$U$	..	0.190	0.487	0.460	0.221	0.281	0.619
$T$ , $10^9$ years	5.13	18.5	18.5	7.24	10.9	16.1	30.5
$f$	..	0.244	0.454	0.241	0.260	0.301	0.291

in 300 million years; allowing for undiscovered cases, there should be at least one impact upon Mars of an asteroid with  $d > 10$  km each  $10^8$  years, and much more impacts of smaller ones—as a guess, one of  $d > 1$  km each  $10^5$  years and one of  $d > 0.1$  km each one hundred years;  $d = 0.1$  km may correspond more or less to the meteor crater of Arizona (8); it seems really worth while looking for meteor craters and meteor impact on Mars! With the scarcity of water and the lesser density of the Martian atmosphere, erosion should proceed there much slower than on earth; the traces of impact should stay at least for 10 million years and, perhaps, ten times longer than that, in which case the surface of Mars should be covered with hundreds of thousands of meteor craters exceeding in size the Arizona crater. It is not impossible that certain characteristic features of the Martian topography such as the spots called “*lacus*” or “*lucus*” may be related to past impacts of asteroids.

For Pluto, form. (45) and Table I give  $T_{\max} = 33 \times 10^9$  years,  $q_{\min} = 8.48$  a.u.; under exceptional circumstances close encounters with Neptune may cause Pluto to cross the orbit of Saturn, but  $T$  will always remain within the same order of magnitude. For Eros, cumulative perturbations at close encounters with Mars may lead to the extreme values  $T_{\max} = 3.45 \times 10^9$  years,  $q_{\min} = 0.52$ ,  $q'_{\max} = 7.6$  a.u. Of the 27 asteroids of the Martian group, 16 have  $\bar{U} > \sqrt{2} - 1$  and may thus be thrown out of the solar system by the cumulative action of Mars alone; however, this action is rather slow and hardly compares to the chances of physical collision, owing to the small relative mass of Mars.

The asteroids of Table IV show a crowding of their perihelia toward the outer margin of the Martian orbit:

limits of perihelion	1.676-	1.599-	1.523-	1.447-	<	
distance	-1.600	-1.524	-1.448	-1.372	1.372	All
number	12	5	1	4	5	27

This may merely be the result of an increase of the asteroid density outwards.

(b) *Crossings with several planets.* The probabilities of collision being small, the sum  $P = \sum P_i$  of the separate probabilities of collision with individual planets is used in (24). Tables V and VI contain the results.

$$p = P_i / \sum P_i \tag{49}$$

is the relative probability of collision with a particular planet. From a comparison of the values of  $p$  we notice that among the inner planets Venus is most efficient in capturing; Mercury and the earth are more or less equal for similar crossings (*cf.*  $Q$ , Table III), whereas Mars is rather inefficient. Among the outer planets Jupiter, of course, dominates; Saturn is about as efficient as the earth, whereas Uranus and Neptune are relatively insignificant. Mars is omitted from Table VI as unimportant. The age expectation

in all cases of the multiple crossings except the Perseids is small as compared with a standard of  $3 \times 10^9$  years; thus, for Apollo the probability of survival  $e^{-t/T}$  within  $3 \times 10^9$  years is  $10^{-18}$ , for a Geminid  $6 \times 10^{-6}$ , for Halley's

TABLE V.

Multiple crossings with inner planets.

Object	Apollo	Adonis	Hermes	Baade's 1949 MA	Encke's Comet (Taurids)	Geminids
$d$ , km ..	1.0	1.3	0.4	1.4	—	—
$T$ , $10^6$ years	72	72	42	170	270	250
Mercury $\left\{ \begin{array}{l} U \\ p \end{array} \right. \dots$	—	0.459	—	0.979	0.581	1.10
	—	0.02*	—	0.22	0.28	0.23
Venus $\left\{ \begin{array}{l} U \\ p \end{array} \right. \dots$	0.400	0.754	0.319	1.040	0.924	1.17
	0.66	0.43	0.66	0.45	0.43	0.45
Earth $\left\{ \begin{array}{l} U \\ p \end{array} \right. \dots$	0.574	0.856	0.153	1.004	1.000	1.16
	0.30	0.51	0.31	0.29	0.26	0.28
Mars $\left\{ \begin{array}{l} U \\ p \end{array} \right. \dots$	0.597	0.896	0.455	0.827	1.029	1.07
	0.04	0.04	0.03	0.04	0.03	0.04
Reference	(7)	(7)	(7)	(9)	(10, 12)	(11, 12)

\* Partial crossing,  $f = 0.14$ .

comet or an Orionid meteor 0.008, for a Perseid 0.35. The actual life-time must be even shorter because of the probability of being "thrown out" in a close encounter which, in the case of Jupiter, may be of the order  $10^{-3}$  per revolution (6), requiring a much shorter life-time than  $T$  in the case of Table VI; for the objects of Table V, however, physical collisions determine the life-time more closely.

One way of interpreting the mere existence of all the short-lived objects listed in Tables V and VI is to regard them as remnants of an originally much more densely crowded medium; in view of the "sweeping" efficiency of the planets, the density of the original medium  $3 \times 10^9$  years ago must then have been from  $10^6$  to  $10^{18}$  times the present density. This, however, appears to be improbable; not only is the original density arrived at too high, but, as shown in the next section, between the planetary orbits a considerable fraction of the original diffuse coarse\* matter is expected to have been preserved at a not much decreased density; the apparent absence of asteroidal bodies from the inner portions of the solar system, as well as the direct photometry of the zodiacal light contradict the existence of such interplanetary remnants (cf. Section 7).

More plausibly it is assumed that these objects have arrived at their present orbits recently; some (Hermes) may have originated from the Mars group of asteroids through close encounters with Mars (cf. Sections 5 and 6); some, particularly the short-period comets and their meteor streams, may be captured, chiefly by Jupiter, from a general parabolic field; their

\* For which the Poynting-Robertson effect is insignificant.

present frequency may be the result of statistical equilibrium between the inward "diffusion" from the parabolic field and the loss from collisions and close encounters (6). The above considerations have no bearing on the

TABLE VI.

*Multiple crossings with outer planets.*

Object	Comet Giacobini-Comet 18661 Comet Halley Comet 1862111				
	944 Hidalgo	Zimmer (Giacobinids)	Tempel (Leonids)	( $\eta$ Aquarids, Orionids)	Tuttle (Perseids)
$T$ , $10^8$ years	0.78	0.185	2.22	6.2	28.6
Venus	$\left\{ \begin{array}{l} U \\ p \end{array} \right.$	—	—	2.32 0.10	—
Earth	$\left\{ \begin{array}{l} U \\ p \end{array} \right.$	0.694 0.02	2.35 0.15	2.23 0.04	2.01 0.12
Jupiter	$\left\{ \begin{array}{l} U \\ p \end{array} \right.$	0.966 0.71	0.729 0.98	1.91 0.70	1.90 0.72
Saturn	$\left\{ \begin{array}{l} U \\ p \end{array} \right.$	0.747 0.29*	—	1.71 0.13	1.77 0.12
Uranus	$\left\{ \begin{array}{l} U \\ p \end{array} \right.$	—	—	1.32 0.02	1.55 0.01
Neptune	$\left\{ \begin{array}{l} U \\ p \end{array} \right.$	—	—	—	1.30 0.01

\* Partial crossing,  $f = 0.5$ .

apparent decay of comets, or the dispersion of meteor streams which refer more to the conditions of observability than to the actual disappearance of the constituent particles.

(c) *The Trojan planets.* The stability of these bodies librating around the Lagrangean point cannot be proved theoretically, if perturbations by Saturn and the other planets over long periods of time are considered. Nevertheless, a *de facto* indication of stability can be inferred from their mere existence in the light of the theory of collisions. If in the course of time a Trojan leaves the Lagrangean point so that its mean longitude will not keep within a limited range from Jupiter, a collision becomes possible the probability of which may be well estimated by assuming the present orbital elements ( $e$ ,  $i$ ) and  $A = 1$ . The relevant figures are contained in Table VII. In all cases the life-time from collisions is of the order of  $10^6$  years and, when perturbations at close encounters are considered, actually much shorter. Further, the Trojans are quite large bodies as compared with the bulk of the asteroids; bodies of this size are comparatively rare among the asteroids and their presence in a considerable number is hardly suggestive of them being remnants of an originally very much larger population. It seems, therefore, that the Trojans have never been subject to a chance of a close approach to Jupiter, in other words, that their present orbits are

actually stable for time intervals of the order of  $3 \times 10^9$  y; or more precisely, if they happened to leave the Lagrangean point, they must have soon returned to it, the time of "wandering" being shorter than a  $10^{-3}$  fraction of their entire existence.

TABLE VII.

*Would-be expectation of life for the Trojans.*

	588	617	624	659	884	911
Object	Achilles	Patroclus	Hektor	Nestor	Priamus	Agamemnon
$d$ , km ..	130	280	210	120	150	180
$T$ , $10^6$ years ..	1.7	5.5	1.2	0.45	0.98	2.8
$U$ ..	0.252	0.409	0.351	0.159	0.211	0.406
	1143	1172	1173	1208	1404	1437
Object	Odysseus	Aeneas	Anchises	Troilus	Ajax	Diomedes
$d$ , km ..	150	120	110	100	90	130
$T$ , $10^6$ years ..	0.01	1.7	0.81	7.1	3.1	1.6
$U$ ..	0.104	0.281	0.194	0.602	0.365	0.381

7. *Sweeping Effect of the Inner Planets and Arrangement of Planetoidal Matter.* The collision efficiency of the inner planets with respect to particles belonging to the same region of space ( $A$  near 1) is illustrated by Table VIII.

TABLE VIII.

$a$	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
$\sin i$ ..	.02	.05	.10	.20	.80	.40	.60	.80
$e$ ..	.02	.05	.10	.20	.25	.40	.60	.80
Planets ..	Earth	=	=	=	Earth	Earth	and	Venus
$T$ , $10^6$ y.	0.45	2.8	16.5	90	320	85	195	320
$a$ ..	0.9	1.25	1.25	0.723	0.723	0.387		.387
$\sin i$ ..	.80	.20	.40	.02	.80	.02		.80
$e$ ..	.15	.40	.40	.02	.35	.02		.80
Planets ..	Earth	Earth and Mars			Venus			Mercury
$T$ , $10^6$ y.	110	250	410	0.90	165	44		230
$a$ ..	1.524	1.524	1.524		1.524			1.524
$\sin i$ ..	.02	.05	.10		.20			.80
$e$ ..	.02	.05	.10		.20			.30
Planets ..	Mars	=	=		=			Mars
$T$ , $10^6$ y.	300	420	850		1950			6500

It is apparent from the table that particles with persistent orbital dimensions, confined to the inner portions of the solar system and crossing the orbits of one of the three innermost planets must have been practically wiped out during a time interval of  $3 \times 10^9$  years, even in the case of the

largest inclinations and eccentricities ; of those crossing the orbit of Mars, however, a considerable fraction may have survived as actually substantiated by the list of Table IV. Considering the original coarse-grained planetoidal matter to have moved chiefly in orbits of small inclination and eccentricity, vacancies in it must have been created by now around the planetary orbits which are practically complete except near Mars. The resulting distribution of matter depends upon the original distribution of the eccentricities which in a long run determines the possibility of a crossing.

To get at least a qualitative picture of the resulting distribution, we assume the original distribution of the eccentricities as for the asteroids (smoothed and idealized) ; let  $f(e)$  be the assumed frequency function ; the cumulative frequency is  $F(e) = \int_0^e f(e) de$ , and the median  $\bar{e}$  of the eccentricity exceeding a certain fixed value  $e$  is defined by

$$\int_e^{\bar{e}} f(e) de = \int_e^1 f(e) de. \quad (50)$$

The actually adopted figures are given in Table IX ; \*

TABLE IX.

$e$	..	0.00	.02	.04	.06	.08	.10	.12	.14	.16	.18
$F(e)$	..	0.000	.031	.072	.122	.194	.307	.430	.543	.632	.701
$\bar{e}$	..	0.132	.135	.139	.148	.152	.167	.184	.202	.215	.227
$e$	..	.20	.22	.24	.26	.28	.30	.32	.34	.40	0.54
$F(e)$	..	.766	.831	.886	.926	.950	.965	.976	.985	.998	1.000
$\bar{e}$	..	.239	.255	.274	.297	.318	.334	.348	.362	.420	0.560

as to the inclination, an average value  $i_1 = 9^\circ.5$  was assumed ; with this and Table III, the constants to be used in formulae (23), (25') and (21') are :

TABLE X.

Crossing Planet	Mercury	Venus	Earth	Mars
$\sin i$	..	.216	.176	.168
$\frac{1}{2} e_0^2$	..	.0178	.0002	.0044
$a_0 (1 - e_0)$	..	0.310	0.709	0.980
$a_0 (1 + e_0)$	..	0.464	0.737	1.020
				1.372
				1.676

For the eccentricity of the particle the median value,  $\bar{e}$ , for each crossing is taken from Table IX. The last two lines of Table X represent the assumed range in distance swept by the corresponding planet, allowance being made for a secular variation of the eccentricity.

\* Corresponding to an old statistics by Elva Utzinger for the first 809 asteroids, but practically representing also the later data.

Consider an aggregate of particles with a fixed value of the semi-major axis  $a$  and a distribution of the eccentricities as in Table IX; the eccentricity  $e'$  of those crossing the orbit of a planet ( $a_0, e_0$ ) must satisfy the condition

$$a(1 + e') \geq a_0(1 - e_0) \quad \text{when } a < a_0(1 - e_0),$$

or

$$a(1 - e') \leq a_0(1 + e_0) \quad \text{when } a > a_0(1 + e_0).$$

Thus the proportion of crossing particles is a fraction  $1 - F(e')$  of the total number of particles with the given  $a$ ; the probable fraction surviving from collisions after an interval  $t$  is then

$$\chi(a) = F(e') + [1 - F(e')] e^{-t/T}, \quad (51)$$

where  $T$  is calculated assuming the average values  $\bar{e}$  and  $\sin i$  from Tables IX and X as arguments. For multiple crossings with marginal eccentricities  $e', e'', \dots$  the formula is

$$\chi(a) = F(e') + [F(e'') - F(e')] e^{-t/T_1} + [F(e''') - F(e'')] e^{-t/T_2}, \quad (51')$$

where  $T_1$  refers to the single crossings,  $T_2$  to the double crossings, etc. Actually only two planets had to be taken into account at once. For the three innermost planets, at  $t = 3 \times 10^9$ , the computations are very much simplified, as  $e^{-t/T}$  is practically zero in these cases. Table XI and Figure 3 contain the result. The resulting distribution shows a number of peaks corresponding to the space between the planetary orbits, separated by

TABLE XI.

Survival fraction,  $\chi(a)$ , for large particles of an asteroidal distribution of eccentricities;

$$t = 3 \times 10^9 \text{ years}; i = 9^\circ.5;$$

the Poynting-Robertson effect is neglected.

$a$	..	0.225	0.250	0.275	0.300	0.310 - 0.464	0.50	0.55	0.586
$\chi(a)$	..	0.995	0.886	0.516	0.058	$10^{-24}$	0.165	0.614	0.792
$a$	..	0.600	0.650	0.700	0.709 - 0.737	0.750	0.800	0.858	0.900
$\chi(a)$	..	0.707	0.256	0.020	$10^{-59}$	0.025	0.190	0.552	0.245
$a$	..	0.950	0.980 - 1.020	1.050	1.100	1.150	1.196	1.200	1.250
$\chi(a)$	..	0.056	$10^{-30}$	0.051	0.170	0.385	0.565	0.557	0.333
$a$	..	1.300	1.372	1.524	1.676	1.800	2.000	2.800	
$\chi(a)$	..	0.175	0.097	0.150	0.202	0.398	0.743	0.999	

vacancies in the neighbourhood of these orbits; only for Mars the vacancy is not complete, from 10 to 20 per cent. of the asteroidal bodies surviving

in the vicinity of Mars; this, together with the skewness of the survival fraction near the orbit of Mars is well supported by the observational data.

The calculated survival fraction refers to the distribution of the major axes which, for orbits of small eccentricity should be very close to the average space-distribution of the particles.

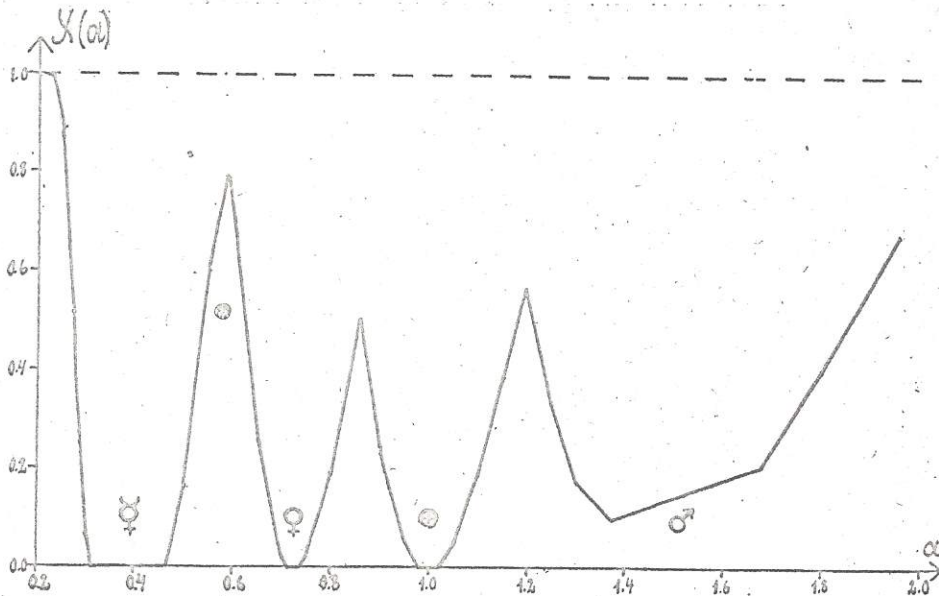


FIG. 3.—Survival fraction (ordinates) in  $t = 3 \times 10^9$  years as function of semi-major axis (abscissae), for an asteroidal distribution of eccentricities and  $i = 9^\circ.5$ .

As stated above, if the present Apollo group of planetoids represents remnants of a primordial asteroidal population, the space between the inner planets should be crowded from  $10^6$  to  $10^{18}$  times more densely with asteroids moving on less eccentric orbits; the light reflected by these should be prominent in the photometry of the zodiacal light.

Fig. 4 represents the surface brightness of such a hypothetical medium, calculated with a uniform scattering in all directions for two cases: I, for a density of the medium  $\rho I = r^{-1} \times \text{const}$ , and II, for an original density as I multiplied by the survival fraction, thus  $\rho II = \rho I \cdot \chi(a)$ . The distance limit for the numerical integrations was set at  $r = 2$  a.u.

Curve II, the planetoidal medium transformed by the sweeping action of the planets, requires a very uneven, wavy slope of the surface brightness; such a feature would have been observable even with the naked eye. Photometric observations certainly do not indicate such an undulatory pattern in the surface brightness of the zodiacal light. If the photoelectric observations at McDonald Observatory (14), made with a field of about  $9^\circ$  in diameter, lack somewhat in resolving power to be decisive in this question, the photographic photometry by Sandig (15) is quite conclusive in denying

the zodiacal light any resemblance to the terrace-shaped curve II of Fig. 4; the actual run of the surface brightness of the zodiacal light along its axis resembles closely the smooth run of I, Fig. 4, with a slightly steeper gradient. Sandig's single exposures, measured over intervals of  $3^{\circ}.24$ , show even a smoother run than his combined curve where zero-point displacements between the individual plates produce a slight unevenness.

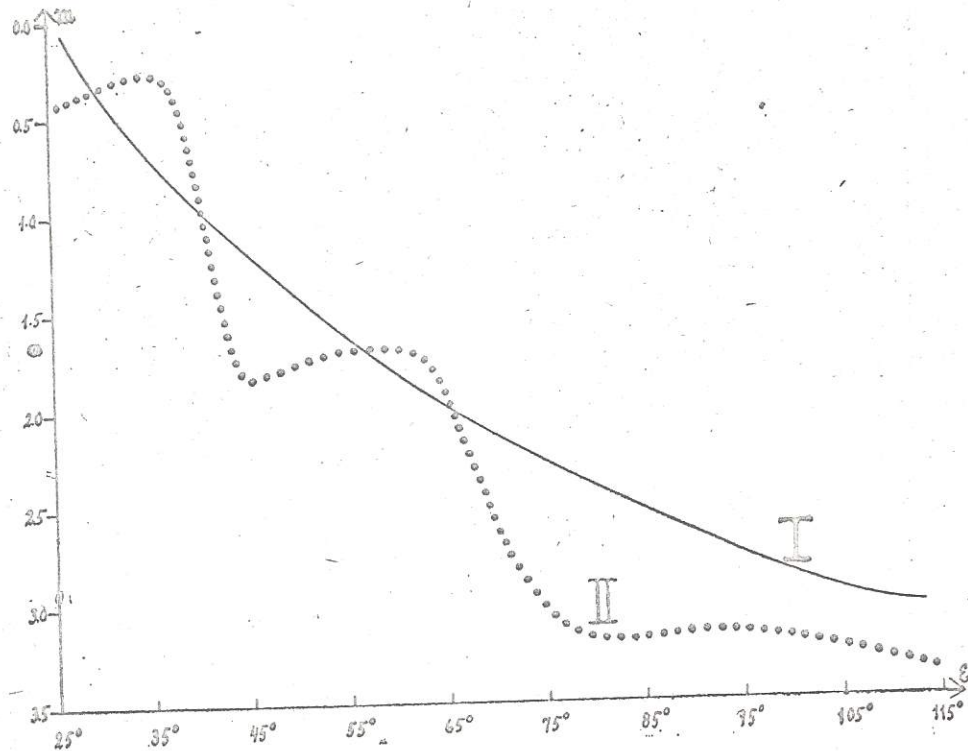


FIG. 4.—Surface brightness (ordinates, stellar magnitudes) of a diffuse medium consisting of large particles, depending upon the apparent elongation from the sun (abscissae) as seen from the earth (at mean distance).

I, full curve: density law  $1/r$ .

II, dotted curve: same density law multiplied by the survival fraction, as in Fig. 3.

The observations thus quite definitely indicate the absence of an appreciable planetoidal component in the zodiacal light. That the absence is not due to unaccounted for perturbations by the planets but appears to be an initial feature of the solar system is borne out by the characteristic distribution of the asteroids in the vicinity of Jupiter's orbit. In spite of the much more powerful perturbing action of Jupiter, the aphelia of the minor planets are crowding quite closely toward Jupiter's orbit and stop almost abruptly near  $r = 4.94$  or at Jupiter's perihelion distance. Table XII contains the relevant data.

Excluding 944 Hidalgo whose orbit is cometary and does not belong to the group at all, the distribution of the aphelia is as follows:

aphelion dist.	4.00-4.18	4.19-4.37	4.38-4.56	4.57-4.75	4.76-4.94	>4.94
frequency	16	4	5	10	4	0

TABLE XII.

Minor planets (except the Trojans) with aphelia  $\geq 4.00$  a.u. (?).

Planet	$a(1+e)$	Planet	$a(1+e)$	Planet	$a(1+e)$
76 Freia ..	4.06	794 Irenaea ..	4.06	1202 Marina ..	4.74
153 Hilda ..	4.59	814 Tauris ..	4.12	1212 Francette ..	4.68
190 Ismene ..	4.61	886 Washingtonia..	4.05	1256 Normannia	4.14
225 Henrietta ..	4.32	944 Hidalgo ..	9.58	1268 Libya ..	4.35
279 Thule ..	4.43	958 Asplinda ..	4.66	1269 Rollandia ..	4.23
319 Leona ..	4.15	965 Angelica ..	4.01	1345 Potomac ..	4.66
334 Chicago ..	4.11	1006 Lagrangea ..	4.26	1362 Griqua ..	4.40
361 Bononia ..	4.77	1036 Ganymed ..	4.10	1373 1935 QN ..	4.50
409 Venusia ..	4.84	1038 Tuckia..	4.88	1439 Vogtia ..	4.40
525 Adelaide ..	4.58	1099 Pigneria ..	4.08	1474 1935 QY ..	4.07
680 Geneveva ..	4.05	1144 Oda ..	4.09	1477 Bonsdorffia	4.09
747 Winchester ..	4.02	1162 Larissa ..	4.43	1512 1939 FE ..	4.63
748 Simeisa ..	4.65	1180 Rita ..	4.69	1529 1938 BC ..	4.76
778 Theobalda ..	4.03				

If there is a trend in this distribution, it seems to reflect a selection effect together with a certain initial uneven distribution; no definite decrease in the frequency of the aphelia as Jupiter's perihelion is approached can be inferred from these data. Thus, the perturbations by as large a mass as Jupiter's have not much influenced the distribution, the vacancy in the asteroidal medium near Jupiter's orbit being practically confined to the region where physical collisions are possible. The largest aphelion in the list is 4.88, by only 0.06 a.u. short of Jupiter's perihelion; if this represents the range at which perturbations become statistically efficient, for another planet the range may be set proportional to  $(am^{1/3})$  which for the earth gives 0.002 a.u., an insignificant figure as compared with the width of the zone of physical collisions (0.04 a.u., Fig. 3). Thus, the distribution of the aphelion distances of the minor planets in the vicinity of Jupiter's orbit represents a check upon the practical validity of the theory of physical collisions as the main agent determining the survival of these bodies.

8. *The Poynting-Robertson Effect.* The quanta of radiation emitted by the sun are carrying a purely radial momentum; therefore, when absorbed (or reflected and scattered) by a particle revolving around the sun, these quanta, in addition to the commonly recognized radial pressure of light will produce a tangential dragging force similar to the action of a resisting medium; the angular momentum per unit mass of the particle will decrease and the

particle will spiral into the sun at a rate depending upon the ratio of surface to mass, thus inversely proportional to the linear dimension of the particle. The absorbed energy being supposed to be re-radiated to space evenly in all directions (with reference to the moving particle), no recoil effect of the own radiation of the particle does occur. A secular decrease of the major axis and of the eccentricity of the orbit of the particle results. The effect is connected with Poynting's name (1903), although his reasoning was based on a misconception of absolute motion, and his predicted effect amounted to only one-third of the correct value. Improvement in the theory was introduced by Larmor. The problem has been more recently revived by H. N. Russell and thoroughly investigated by Robertson (21). For the sake of brevity we will refer to it below as the Robertson effect.

Neglecting the radial component of radiation pressure which, in the solar system, becomes important only for particles of a radius less than  $10^{-4}$  cm, the chief effect amounts to a secular decrease in the major axis of the orbit of the particle,

$$-\frac{da}{dt} = 2ka^{-1}. \quad (52)$$

For the actual solar mass and radiation, and a black (absorbing) particle,

$$k = \frac{3.5 \times 10^{-8}}{\rho r}; \quad (53)$$

where  $\rho$  is the density of the particle in gr/cm<sup>3</sup>,  $r$  its effective radius in cm,  $a$  being given in astronomical units and  $t$  in years.

Integration of (52) yields

$$a_2^2 - a_1^2 = -4k(t_2 - t_1). \quad (54)$$

The eccentricity diminishes at the same time.

As the result of the Robertson effect, all the members of the solar system are drifting toward the sun with different speed; for bodies of planetary dimensions the effect is negligible, but for meteoric particles it becomes rather important. From the standpoint of the theory of collisions the effect amounts to a limitation of the time interval,  $\Delta t$ , during which physical collisions are possible; for a circular orbit of the particle, according to (52)

$$\Delta t = \frac{a_0 \Delta a}{2k}, \quad (55)$$

where  $\Delta a = 2a_0 e_0$  (cf. Table III). The survival fraction for particles of a definite size, drifting through the "sphere of collisions" of a given planet, is then evidently

$$\chi = e^{-\Delta t/T}, \quad (56)$$

where  $T$  is the life-time from collisions (equ. (24)). When  $\Delta t < t_0$ ,  $t_0 = 3 \times 10^9$  y, formula (56) actually applies; when  $\Delta t > t_0$ ,  $\chi = e^{-t/t_0}$  must be taken as in the preceding section.

The calculations are contained in Table XIII.

TABLE XIII.

Survival fraction ( $\chi$ ) for Robertson drift through the spheres of action of the planets;  $\rho = 3 \text{ gr/cm}^3$ ; an asterisk means  $\Delta t > 3 \times 10^9$ , or that the Robertson effect is unimportant.

$r$ , cm	$10^{-3}$	$10^{-2}$	$10^{-1}$	1	10	$10^2$	$10^3$	$10^4$	$T$ , years
Jupiter	0.9	0.31	$10^{-5}$	$10^{-50}$	$10^{-500}$	*	*	*	$1 \times 10^8$
Mars	1	1	1	0.98	0.86	0.21	*	*	$1.3 \times 10^9$
Earth	1	1	1	0.94	0.66	0.015	$10^{-18}$	*	$4 \times 10^7$
Venus	1	1	1	0.99	0.90	0.34	$10^{-5}$	*	$8 \times 10^7$
Mercury	1	1	1	0.98	0.78	0.08	$10^{-11}$	*	$1 \times 10^8$

The assumed values of  $T$  correspond to an average inclination about  $10^\circ$  and to a small eccentricity.

From the table we infer that ordinary meteoric particles and dust,  $r \leq 10$  cm, are little influenced by collisions with the four inner planets: the Robertson drift is too fast for these. Only for  $r > 100$  cm the survival fractions become small and the drift itself small, too; only for these larger particles the calculations of the preceding section, with the Figures 3 and 4, apply. The barrier of Jupiter is more severe; of the particles drifting through Jupiter's orbit only the smallest ones,  $r \leq 10^{-2}$  cm survive in an appreciable proportion.

On the other hand, the range of distance covered by the particles since their origin depends upon their size; for an age of  $t_0 = 3 \times 10^9$  years and an initial distance  $a_1 = 5$  corresponding to Jupiter's orbit the present semi-major axes of particles of various sizes are found as follows (eq. (54) with  $\rho = 3 \text{ gr/cm}^3$ ):

$r$ , cm	$\leq 5.6$	6	10	100
$a_2$	0	1.3	3.3	4.86

Thus, all meteoric particles of the asteroidal complex below a sharp margin near  $r = 6$  cm, encountered in the inner portions of the solar system, if existing for  $3 \times 10^9$  years must have passed through the sphere of action of Jupiter; therefore, according to Table XIII, asteroidal meteors of  $6 > r > 10^{-2}$  cm must be practically absent near and between the four inner planets. On the other hand, the occurrence of dust particles of  $r = 10^{-3}$  cm and less is favoured, a lower margin being set, of course, by the radiation pressure (at about  $r = 2 \times 10^{-5}$  cm.).

Thus, from a combination of the Robertson effect with collisions it appears that in an asteroidal-planetoidal component of the zodiacal light a particle size near  $10^{-3}$  cm might well be predominant, with an almost complete

absence of the intermediate sizes from 0.03 to 6 cm. This lends unexpected support to van de Hulst's and Allen's treatment of the coronal photometric data. Particles of later origin and of intermediate sizes may be, of course, present.

For  $t_0 = 3 \times 10^9$  y. and  $a_2 = 1$ , thus for asteroidal objects crossing at present the orbit of the earth, the distance of origin,  $a_1$ , is found from (54) as follows ( $\rho = 3$  gr/cm<sup>3</sup>):

$r$ , cm	$10^{-3}$	$10^{-2}$	$10^{-1}$	1	5.9	10	$10^2$	$10^3$	$10^4$
$a_1$	360	120	37	12	5	3.9	1.55	1.067	1.007

Of these, the sizes from  $r = 10^{-1}$  (ordinary visual meteors) to  $r = 6$  cm (bright fire-balls) are cut off by Jupiter; those around  $r = 10$  cm originate from the densest regions of the asteroidal ring and may be expected to be rather numerous; the larger sizes should be comparatively rare again (cf. Table XIII). Therefore, there is no reason to be perplexed by the apparent failure to observe visually the numerous meteors of the zodiacal-light cloud, even if the interpretation given to it by Allen and van de Hulst is correct; a crucial test might consist in the observation of the faintest telescopic meteors around the 16th–20th magnitude, which should be unusually numerous if the coronal theory is valid.

9. *The Yarkovsky Effect.* A specific effect connected with radiation phenomena in a rotating particle consists in a recoil pressure excess of the radiation emitted by the "evening" hemisphere over the "morning" hemisphere; a very slight excess of the evening over the morning temperature, of the order of  $10^{-2}$  deg K at  $a = 1$ , can produce a drag of the same order as the Robertson effect. The effect has been considered by Civil Engineer Yarkovsky, in a pamphlet published in Russian at St. Petersburg about 1900; the writer read the pamphlet about 1909 and can refer to it only from memory.

Yarkovsky considered the aerodynamic resistance to planetary motions of a World Ether, in which respect his conclusions are, of course, untenable. But his other effect, produced by the uneven emission of radiation by a rotating body in which a phase lag of surface temperature occurs, may be important. When rotation and orbital motion are in the same direction, the Yarkovsky effect produces a positive drag opposite to the Robertson effect and may counterbalance it; when the sense of rotation is opposite to orbital motion, both effects add working in the same direction.

The ratio of the Yarkovsky to the Poynting–Robertson drag may be set equal to

$$\frac{y}{P_r} = -\frac{c}{v} \cdot \frac{2}{3} \frac{\Delta\theta}{\theta} \cdot \cos \epsilon, \quad (57)$$

where  $c$  is the velocity of light,  $v$  the orbital velocity (circular case),  $\theta$  the

absolute temperature,  $\Delta\theta$  the absolute difference of surface radiative temperature between the evening and morning points at "equinox,"  $\epsilon$  the inclination of the equator of rotation to the plane of the orbit.

For a stony sphere, with proper values for the conductivity, specific heat and the solar constant dimensional considerations yield

$$\Delta\theta = \frac{1}{6a^2} \sqrt{P} \text{ (deg K)} \quad (58)$$

in the solar system;  $P$  is the period of axial rotation of the particle in seconds,  $a$  the semi-major axis of its orbit in astronomical units. The validity of (58) is restricted to the condition

$$r > \frac{1}{5} \sqrt{P}, \quad (59)$$

which means that the "daily" temperature oscillations are dying out well outside the centre of the particle.

When (59) is not fulfilled, the order of magnitude of  $\Delta\theta$  is given by

$$\Delta\theta = \frac{5}{6} \frac{r}{a^2}, \quad (58')$$

for

$$r < \frac{1}{5} \sqrt{P}. \quad (59')$$

Substituting these formulae into (57), we obtain

$$\frac{y}{P_g} = -3.97 \cos \epsilon \cdot \frac{\sqrt{P}}{a} \quad (60)$$

in the first case, and

$$\frac{y}{P_g} = -19.8 \cos \epsilon \cdot \frac{r}{a^2} \quad (60')$$

in the second case.

In the case of retrograde rotation ( $180^\circ > \epsilon > 90^\circ$ ) the Yarkovsky effect adds to the Robertson effect, leading to an accelerated rate of decrease of the major axis.

In the case of direct rotation ( $0^\circ < \epsilon < 90^\circ$ ) the effects are opposite in sign and the rate of the secular variation,  $da/dt$ , is decreased or even inverted: the particles may spiral outward. For a given period of rotation or radius the ratio of the two effects depends upon  $a$ ; there exists a certain average distance,  $a_0$ , at which the ratio is  $-1$  and where the two effects mutually cancel.

Meteor particles in space are expected to possess a rapid rotation, produced by collisions with smaller dust particles; for an age of  $3 \times 10^9$  years the writer has estimated (3) the order of magnitude of the average period of rotation as

$$\bar{P} = \frac{r^{3/2}}{1800} \text{ (sec)}, \quad (61)$$

valid for  $0.01 < r < 5$  cm, and as

$$\bar{P} = \frac{r}{18000} \quad (61')$$

for  $r < 0.01$  cm.

Even assuming that these periods are underestimated by a factor of 10 (if the age and the frequency of collisions are overestimated, both entering in a power of 1/2), we find  $r^{1/5} \sqrt{\bar{P}} = 73 r^{1/4} > 1$  for all possible radii exceeding 0.01 cm and  $r^{1/5} \sqrt{\bar{P}} = 215 r^{1/2} > 1$  valid down to  $r = 2 \times 10^{-5}$ . Therefore the first case of equations (59), (58) and (60) must be overwhelmingly frequent. For a tenfold period as compared with (61) or (61'), (60) yields

$$\frac{y}{P_0} = -0.3 \cos \epsilon \cdot \frac{r^{3/4}}{a} \text{ for } r > 0.01 \text{ and } -0.09 \cos \epsilon \cdot \frac{r^{1/2}}{a} \text{ for } r < 0.01.$$

For  $\cos \epsilon = \frac{2}{3}$  the equilibrium distance where this ratio equals -1 is as follows:

$r$ , cm	$10^{-3}$	$10^{-2}$	$10^{-1}$	1	3	10
$a_0$ , a.u.	(0.002)	(0.006)	(0.04)	0.2	0.5	1.1

The minimum distance of a solid particle from the sun can hardly be less than 0.01 a.u.; therefore the above equilibrium distances,  $a_0$ , have a real meaning only in the case of the larger particles,  $r = 1 - 10$  cm. Only for such particles (belonging to the class of "fireballs") the Yarkovsky effect may play a statistically important rôle. As stated above, particles around 10 cm. radius are likely to have drifted from the midst of the asteroidal ring toward the earth's orbit; in the case of direct rotation the Yarkovsky effect seems to be able to check the drift also somewhere in the neighbourhood of the earth's orbit and to force such particles to move outward from the inner portions of the solar system. It is therefore likely that a sifting out and accumulation of directly rotating particles of a definite size proceeds at a definite distance from the sun. The particles with retrograde rotation are absorbed into the sun at an accelerated rate. The whole phenomenon is more complicated because the elements of rotation of a particle are subject to discontinuous changes from collisions. Also, the Robertson effect exerts a retardation of axial rotation; the period of rotation is reduced in the ratio

$$\frac{P_2}{P_1} = \sqrt{\frac{a_2}{a_1}},$$

$a_2$  and  $a_1$  satisfying equation (54).

For the small particles,  $r < 10^{-2}$  cm, which can penetrate the sphere of action of Jupiter and which alone of all the particles with orbits of small eccentricity can participate in producing the observable phenomenon of the F - corona, the Yarkovsky effect is negligible.

10. *The Nature of the Zodiacal Light.* Without a definitive solution being possible at present, the above results help to restrict the diversity of hypotheses advocated to explain the phenomenon of zodiacal light.

As shown above, clouds of large planetoidal particles ( $r > 100$  cm) moving in more or less persistent orbits of small eccentricity between the orbits of the inner planets cannot contribute appreciably to the zodiacal light; otherwise certain vacancies around the orbits of the planets would have become conspicuous in the photometric data; this refers chiefly to the three inner planets and is enhanced by the fact that not a single planetoidal object has yet been discovered having its aphelion inside the orbit of Mars. Around Mars and outside, however, planetoidal matter may be well present, its larger representatives being actually observable (Table IV). The "two-ring" hypothesis of Hoffmeister (16) deserves still attention from this standpoint.

On the other hand, the pronounced concentration of the zodiacal light toward the plane of the ecliptic indicates that its chief component belongs to the solar system and that the average inclination of the orbits of the constituent particles is small. Particles of less than 0.01 cm and those between 6 - 10 cm radius, displaced and sifted out by the Poynting-Robertson, the Yarkovsky effect and by the physical collisions with the planets, chiefly Jupiter, can play a rôle among those moving in orbits of small eccentricity. However, such particles have not been observed individually. The directly observed objects which actually move inside the space of the zodiacal light combine a large eccentricity with small inclination; these simultaneous properties are met in a very pronounced degree in the orbits of the short-period comets, as well as in the objects of Table V:

	Apollo	Adonis	Hermes	1949 MA	Encke's Comet	Geminids
$i$	6°.4	1°.5	4°.7	23°.2	12°.5	23°.5
$e$	0.57	0.78	0.47	0.83	0.85	0.90

To the same class belong the "zodiacal-light meteors" of small heliocentric velocity observed by the rocking mirror which have large orbital eccentricities (17), as well as the daytime meteor streams revealed by radar observations (18). The most proper term for such objects seems to be "zodiacal particles"; it is but natural to assume that the large observable

objects indicate the presence of a medium consisting of particles of all sizes up to the smallest and those most efficient in scattering light; the main component of the zodiacal light may be well due to such zodiacal particles. Moving in orbits of large eccentricity these particles are subject to high chances of physical collisions with the planets and of perturbations at close encounters. Therefore the zodiacal particles have to be considered as transient members in the space they are occupying at present, their collision age being of the order of  $10^8$  years only (Section 6); there must exist a steady supply of them, to replace those which are continually intercepted by the planets. Their combination of large eccentricity with small inclination is highly suggestive of products of close encounters (*cf.* Section 5 and Ref. (6)).

In addition to these zodiacal particles two more components may be present: the cometary component, due to particles related to long-period or nearly parabolic comets, with small or no concentration toward the ecliptic; and the interstellar component.

The expected order of magnitude of the interstellar component can be estimated from the value of the interstellar absorption coefficient, independently of any assumptions regarding the distribution of sizes of the absorbing particles. Direct observations of the surface brightness of galactic dark nebulae indicate a high scattering power, or a high value of the spherical albedo of interstellar matter (14, 19); an albedo between 0.5 and 1.0 is suggested (19) which means that this matter consists chiefly of dielectric particles (ice crystals, mineral dust) and that the rôle of absorbing metal particles must be subordinate. Thus the interstellar matter scatters most of the absorbed light and the amount of scattered light can be estimated fairly well from the absorption coefficient without any knowledge of the diameters and physical properties of the particles.

Assuming a colour absorption of 0.17 mag. per kiloparsec, and a ratio of photographic to colour absorption equal to 6 (20), a plausible value for the general absorption in galactic space of 1.0 mag/kiloparsec or an absorption coefficient  $c = 0.5 \times 10^{-8}$  per astr. unit results. For an unchanged density of the diffuse matter in the vicinity of the sun, a uniform scattering in all directions and an albedo = 1 the surface brightness of scattered solar light as observed from the earth in the direction  $90^\circ$  from the sun is (self-absorption being negligible)

$$L = \frac{cd\omega}{4\pi} \int_0^\infty \frac{dx}{1+x^2}, \quad \text{or with } \frac{d\omega}{4\pi} = \frac{1}{41200}, \quad L = \frac{\pi}{2} \cdot \frac{c}{41200},$$

in units of solar luminosity per square degree, or  $L = 5.1$  mag per square degree; the condensing action of the sun's gravitation may increase the effective density to about the double value, whereas the albedo being less than 1 and the scattering in  $90^\circ$  being less than the average may counter-balance this effect. Thus 5.1 mag (photographic) per square degree remains a fair estimate for the expected intensity of the interstellar component of zodiacal light in  $90^\circ$  from the sun; this may be compared with a figure of

[18]

L. J. B. 1917, p. 156, fig. 1

50 tenth-magnitude stars or 5.8 mag per square degree, found by Elvey and Roach (14) for the intensity of the zodiacal light near the pole of the ecliptic (at 4500 Å); the agreement of the two figures is excellent, for the kind of data so difficult to determine; especially if it is considered that radiation pressure may repel some of the optically most active small interstellar particles. It may be also noted that Sandig's photographic magnitudes (15) are by about one magnitude systematically brighter than those of Elvey and Roach. In any case, it appears to be probable that the residual intensity of zodiacal light in the direction of the pole of the ecliptic is almost entirely due to the interstellar component, i.e. to a cloud of hyperbolic meteors rendered visible in the inner portions of the solar system by the scattering of solar light.

ARMAGH OBSERVATORY,  
December 2, 1949.

REFERENCES.

- (1) C. W. ALLEN 1946 *Monthly Not. R.A.S.*, 103, 137.
- (2) H. C. VAN DE HULST 1947 *Astroph. Journ.*, 105, 471.
- (3) S. NEWCOMB 1894 *Astron. Papers Amer. Ephem.*, 5, Pt. 4.
- (4) J. N. STOCKWELL 1873 *Smithsonian Contrib. to Knowledge*, 13.
- (5) H. A. NEWTON 1893 *Mem. Nat. Acad., Washington*, 6.
- (6) A. J. J. VAN WOERKOM 1948 *Bull. Astr. Inst. Netherl.*, 10, 445.
- (7) Inst. Theor. Astr. of the Acad. of Sc., USSR., *Ephemerids of Minor Planets*, 1948, Pt. II.
- (8) E. ÖPIK 1936 *Tartu Obs. Pub.*, 23, No. 6.
- (9) L. E. CUNNINGHAM 1949 *Circ. Un. Astr. Intern.*, 1226.
- (10) F. L. WHIPPLE 1940 *Harvard Repr.*, 210.
- (11) ——— 1947 *Harvard Repr.*, Ser. II-16.
- (12) C. HOFFMEISTER 1948 *Meteoric Currents*, Weimar.
- (13) E. ÖPIK 1932 *Harvard Repr.*, 79; also J. H. OORT 1950 *Bull. Astr. Inst. Netherl.*, 11, 91.
- (14) C. T. ELVEY, and F. E. ROACH 1937 *Astroph. Journ.*, 85, 213.
- (15) H. U. SANDIG 1939 *Mitteil. Sternwarte Breslau*, 5, 25.
- (16) C. HOFFMEISTER 1932 *Veroff. Berlin-Babelsberg*, 10, No. 1.
- (17) E. ÖPIK 1940 *Tartu Obs. Pub.*, 30, No. 5, 61.
- (18) A. ASPINALL, J. A. CLEGG, and A. C. B. LOVELL 1949 *Monthly Not. R. A. S.*, 109, 352, and earlier publications.
- [19] V. RIVIES 1938 *Tartu Obs. Pub.*, 30, No. 1, VI.
- (20) E. M. LINDSAY 1950 *Nature*, 165, 343, and different other sources.
- (21) H. P. ROBERTSON 1937 *Monthly Notices R.A.S.*, 97, 423.